

Utilizing the Rao-Blackwellization Technique to Improve Ratio Estimators in Adaptive Cluster Sampling

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Abstract In addition to the variable of primary interest, often other auxiliary variables would be available in a practical sampling survey case and one would like to make use of the auxiliary information to improve the inference of the population quantity of interest. Rao-Blackwellization has been used to improve the Hansen-Hurwitz and Horvitz-Thompson univariate unbiased estimators in Adaptive Cluster Sampling by finding the conditional expected value of the original unbiased estimators given the sufficient or minimal sufficient statistic. In principle, the same idea can be used to find better ratio estimators, however, the calculation of taking all the possible combinations into account can be extremely tedious. The simplified analytical forms of such ratio estimators are not currently available. For practical interest, several improved ratio estimators in Adaptive Cluster Sampling are proposed in this article. The proposed ratio estimators make use of the Rao-Blackwellized univariate estimators but not the real Rao-Blackwellized versions of the original ones. The performances are evaluated by simulation study. The simulation result indicates that the proposed ratio estimators are able to provide considerable advantageous estimation results over the original ones.

Key words: Adaptive Cluster Sampling, Ratio Estimator, Rao-Blackwellization, Sufficient Statistic, Minimal Sufficient Statistic, Hansen-Hurwitz Estimation, Horvitz-Thompson Estimation.

1 Introduction

First proposed by Thompson [10], Adaptive Cluster Sampling (ACS) is a data-driven sampling method which can provide higher sampling yields together with better inference regarding the population quantity of interest than the comparable conventional sampling designs when the population of interest is a rare and/or clustered one, in which the conventional designs often fail to fit. Various ACS designs

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associated with different initial designs have been developed in the past, and the designs in the family of ACS have received considerable attention in different real-world survey situations because of their practical flexibility and ability to provide better estimation results (Please see Turk and Borkowski [13] for a detailed review).

Originally the estimators in ACS can be divided into two categories; estimators of the modified Hansen-Hurwitz type (Hansen and Hurwitz [7]) and the modified Horvitz-Thompson (Horvitz and Thompson [8]) type. Both of these estimators are design-unbiased, however still it is possible to further improve these estimators since they are not the function of the minimal sufficient statistic (Thompson and Seber [12]). In addition, the data observed for the edge units which are not initially selected will not be used in these estimators (Thompson [10]), hence part of the observed data is wasted. Salehi[9] and Félix-Medina [6] work out the analytical forms of the Rao-blackwellized unbiased estimators conditioning on the minimal sufficient statistic. For the simpler and more practical versions, Dryver and Thompson [5] proposed other Rao-Blackwellized unbiased estimators conditioning on a sufficient statistic. These Rao-Blackwellized estimators make use of the information obtained from all the edge units, hence all the observed data can be utilized in the Rao-Blackwellized estimators.

All of the estimators described above are univariate estimators that make use of the information from the population variable of interest, denoted as y , itself only. As in the usual survey sampling situations, variables other than y often also are available, and naturally one would like to utilize these auxiliary information for better inference. Ratio estimation is a common estimation method which makes use of the information obtained from a certain auxiliary variable, denoted as x . Dryver and Chao [4] proposed ratio estimators based on the modified Hansen-Hurwitz estimator or modified Horvitz-Thompson estimator. These two ratio estimators, denoted as $\hat{\mu}_{r,hh}$ and $\hat{\mu}_{r,ht}$ are considerably more efficient than other comparable estimators with the existence of certain correlation between x and y . Nevertheless, still the information of the edge units which are not initially selected is ignored. Hence, it is reasonable to assume that they can be improved if this ignored information can be utilized at the inference stage. In this article, several improved ratio estimators which make use of the ignored data will be proposed.

Section 2 will briefly review the typical unbiased estimators, Rao-Blackwellized unbiased estimators, and ratio estimators in ACS proposed in the past. The improved ratio-estimators proposed in this article will be described in Section 3. To evaluate the related performance of the proposed ratio estimators compared to the original ones described in Dryver and Chao [4], results of simulation studies are provided in Section 4. Section 5 gives a summary of this article.

2 Adaptive Cluster Sampling and the Associated Estimators

In ACS, one needs to define the neighborhood of a unit and the condition of interest, C , a function of y . C is often defined as $y > c$ or $y \geq c$ for a pre-specified constant c . The population can be partitioned into K networks, denoted as $\{\Psi_1, \dots, \Psi_K\}$, according to the definitions of C and the neighborhood. Define m_k as the number of units belonging to network Ψ_k , $\sum_{k=1}^K m_k = N$, and w_{yk} and w_{xk} as the average of y -value and x -value in the k^{th} network, respectively. For the general sampling procedure in ACS, please refer to Thompson [10].

2.1 Unbiased Estimators under ACS

Thompson [10] proposed two unbiased estimators in ACS for the population mean of the variable of interest y , $\mu_y = \sum_{i=1}^N y_i/N$; the modified Horvitz-Thompson estimator and the modified Hansen-Hurwitz estimator. Let n_0 be the initial sample size, s_0 be the initial sample, and n_k be the number of times network Ψ_k be intersected by s_0 , the modified Horvitz-Thompson estimator for μ_y is

$$\hat{\mu}_{y.ht} = \frac{1}{N} \sum_{k=1}^K \frac{m_k \cdot w_{yk}}{\alpha_k} \cdot I(n_k > 0) \quad (1)$$

where $\alpha_k = 1 - \binom{N-m_k}{n_0} / \binom{N}{n_0}$ is the initial inclusion probability of the k^{th} network. The modified Hansen-Hurwitz estimator is based on the draw-by-draw selection probabilities that the network a unit belongs to is intersected by the initial sample,

$$\hat{\mu}_{y.hh} = \frac{1}{n_0} \sum_{k=1}^K n_k w_{yk} \quad (2)$$

It can be shown that there is no complete sufficient statistic available under a finite fixed population sampling (Thompson and Seber [12]), in which the population size is finite and the values of the population variable y , $(y_1, \dots, y_N)$, are considered as fixed but unknown constants. However, the minimal sufficient statistic is available and it is the collection of an unordered set of distinct sampled units s_R and the associated observations $\mathbf{y}_{s_R}$, denoted as $d_R = \{s_R, \mathbf{y}_{s_R}\}$. Better estimators can be obtained by conditioning $\hat{\mu}_{y.ht}$ or $\hat{\mu}_{y.hh}$ on d_R . However, the calculation can be very tedious or even impossible to carry out in practice. Salehi [9] and Félix-Medina [6] derived the analytical expressions of the Rao-Blackwellized versions of the modified Horvitz-Thompson and Hansen-Hurwitz estimators conditioning on d_R . In this article, we use Salehi's method to construct the new ratio estimators under ACS, and denote his estimators as $\hat{\mu}_{y.ht(ms)}$ and $\hat{\mu}_{y.hh(ms)}$.

For practical interest, Dryer and Thompson [5] proposed other improved estimators by conditioning $\hat{\mu}_{y.hh}$ and $\hat{\mu}_{y.ht}$ on a sufficient statistic. They divided the final sample into two parts: a core part s_c and the remaining part $s_{\bar{c}}$. s_c is the set of all the distinct units for which condition C is satisfied, and $s_{\bar{c}}$ consists of the distinct units for which condition C is not met. For unit i , let f_i be the number of times that the network to which unit i belongs is intersected by the initial sample. Then, the statistic $d^+ = \{(i, y_i, f_i), (j, y_j); i \in s_c, j \in s_{\bar{c}}\}$ is a sufficient statistic (Dryver and Thompson [5]). We denote the Rao-Blackwellized estimator based on d^+ for μ_y as $\hat{\mu}_{y.ht(s)}$ and $\hat{\mu}_{y.hh(s)}$.

The estimators for μ_x , the population mean of the auxiliary variable x , can be defined similarly.

2.2 Ratio Estimators under ACS

For the sampling situation in which certain auxiliary variable is available, Chao [1] proposed a generalized ratio estimator (e.g. Cassel [2]) by replacing the inclusion probability with the initial selection probability, and this ratio estimator can also be considered as one based on the modified Horvitz-Thompson estimator,

$$\hat{\mu}_{r.ht} = \frac{\hat{\mu}_{y.ht}}{\hat{\mu}_{x.ht}} \cdot \mu_x. \quad (3)$$

In addition, Dryver and Chao [4] proposed a modified ratio estimator for μ_y viewing ACS from the perspective of a transformed population. This modified ratio estimator makes use of the modified Hansen-Hurwitz estimator to estimate μ_y and μ_x ,

$$\hat{\mu}_{r-hh} = \frac{\hat{\mu}_{y-hh}}{\hat{\mu}_{x-hh}} \mu_x, \quad (4)$$

and in fact $\hat{\mu}_{r-hh}$ can be viewed as the traditional ratio estimator in the transformed population (Dryver and Chao [4]).

The ratio estimators proposed by Chao [1] and Dryver and Chao [4] are more efficient than the original univariate unbiased estimators if x and y are correlated to a certain degree. They are also better than the conventional ratio estimators under comparable sample size. However, still these ratio estimators do not use the data obtained from the edge units which are not initially selected, and it is expected that better ratio estimators can be constructed if the data can be better used.

3 Improved Ratio Estimators

One way to carry out the Rao-Blackwellized versions of $\hat{\mu}_{r-hh}$ and $\hat{\mu}_{r-ht}$ is to use the procedure described in Thompson [10]. However, most likely such a procedure cannot be applied in practice because of the tedious computation required. Further, it can be shown the approaches used in Salehi [9] and Félix-Medina [6] cannot be used to carry out the analytical forms of their Rao-Blackwellized versions conditioning on d_R and d^+ . Therefore, in this article we proposed four improved ratio estimators in which the Rao-Blackwellization technique is utilized in a rather straightforward and simple manner. These estimators are

$$\hat{\mu}_{r-ht(ms)} = \frac{\hat{\mu}_{y-ht(ms)}}{\hat{\mu}_{x-ht(ms)}} \mu_x, \quad (5)$$

$$\hat{\mu}_{r-ht(s)} = \frac{\hat{\mu}_{y-ht(s)}}{\hat{\mu}_{x-ht(s)}} \mu_x, \quad (6)$$

$$\hat{\mu}_{r-hh(ms)} = \frac{\hat{\mu}_{y-hh(ms)}}{\hat{\mu}_{x-hh(ms)}} \mu_x, \quad (7)$$

and

$$\hat{\mu}_{r-hh(s)} = \frac{\hat{\mu}_{y-hh(s)}}{\hat{\mu}_{x-hh(s)}} \mu_x, \quad (8)$$

These ratio estimators are not the true Rao-Blackwellized ones, but make use of the Rao-Blackwellized univariate estimators of μ_x and μ_y to estimate the sample ratio. Hence,

$$\begin{aligned} \text{MSE}(\hat{\mu}_{r-ht(ms)}) &\leq \text{MSE}(\hat{\mu}_{r-ht(s)}) \leq \text{MSE}(\hat{\mu}_{r-ht}) \\ \text{MSE}(\hat{\mu}_{r-hh(ms)}) &\leq \text{MSE}(\hat{\mu}_{r-hh(s)}) \leq \text{MSE}(\hat{\mu}_{r-hh}) \end{aligned}$$

can not be assured. Nevertheless, we expect that they would be better in most situations by separately improving the numerator and denominator of the original versions. In the next section, their performance will be evaluated by simulation study

based on populations generated by a linked pairs process (Diggle [3]) together with the Bivariate Poisson cluster process (Chao [1]), which is a generalization of Poisson cluster process (Diggle [3]).

4 Simulation Study

To evaluate the performance of the proposed ratio estimators $\hat{\mu}_{r,(\dots)}$ from a design-based perspective, two different populations are generated based on two set of parameters $(\theta_0, \theta_1, \theta_2)$ of the Bivariate Poisson cluster process, where θ_0 is the number of parents, θ_1 and θ_2 are the numbers of two types of siblings. θ_0 can be considered as the number of clusters, and θ_1 and θ_2 can be viewed as the total of y and x values in the clusters. The study region is a squared one consisting of $30 \times 30 = 900$ squared units, which are the sampling and observational units. The population variable of interest and the auxiliary variable are the number of the two types of siblings in each unit. The first illustrative population in Figure 1 (a) is considered as a rare and clustered population in which the adaptive cluster sampling would be recommended ($\theta_0 = 4, \theta_1 = \theta_2 = 200$), whereas the second one in Figure 2 (a) would be one in which the adaptive cluster sampling might not be an appropriate sampling design ($\theta_0 = 10, \theta_1 = \theta_2 = 150$). The neighboring units are defined as the adjacent units on the left, right, top, and bottom, and the condition of interest is chosen as $C = \{y; y \geq 5\}$.

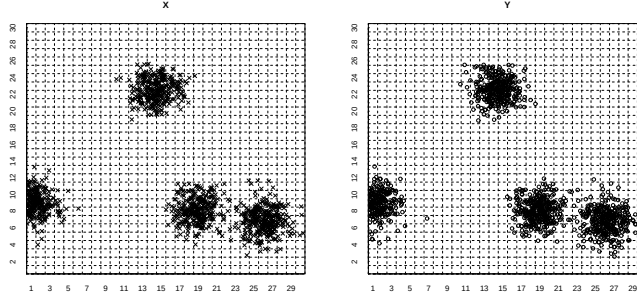
The performance of $\hat{\mu}_{r,(\dots)}$ are compared to the ratio estimators $\hat{\mu}_r$ proposed in Dryver and Chao [4] under different initial sample sizes $n_0 = 1, \dots, 200$ and the conventional ratio estimator $\hat{\mu}_r$ under the associated comparable final sample sizes. $\hat{\mu}_r$ is used as a reference to evaluate the performances of other ratio estimators in ACS. For each n_0 , 5,000 initial sample are selected and $\hat{\mu}_{r,hh}$, $\hat{\mu}_{r,ht}$, $\hat{\mu}_{r,hh(s)}$, $\hat{\mu}_{r,ht(s)}$, $\hat{\mu}_{r,hh(ms)}$, and $\hat{\mu}_{r,ht(ms)}$ are calculated based on the same final sample. In addition, the conventional $\hat{\mu}_r$ is calculated based the sample selected by SRSWOR with the sample size the same as the final sample size in ACS. The evaluation is based on the empirical relative efficiency (E.R.E.) of each $\hat{\mu}_{r,\dots}$ to $\hat{\mu}_r$ under the comparable final sample size.

The E.R.E. of $\hat{\mu}_{r,\dots}$ to $\hat{\mu}_r$ is defined as the ratio of the empirical mean-squared error of $\hat{\mu}_r$ to $\hat{\mu}_{r,\dots}$. Therefore, a value of E.R.E. greater than one indicates that the mean-squared error of $\hat{\mu}_{r,\dots}$ is smaller than $\hat{\mu}_r$ under the comparable sample size. Furthermore, the greater the E.R.E. of $\hat{\mu}_{r,\dots}$ is, the better the performance of $\hat{\mu}_{r,\dots}$ is. The empirical mean-squared error is calculated by taking average of the squared error between the estimates and the true value of μ under each 5,000 samples.

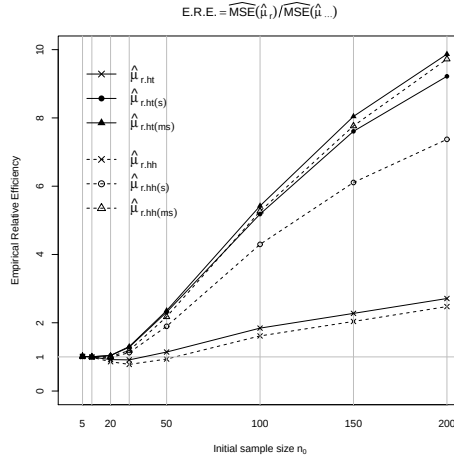
The realized population mean and variance of the population variable of interest y in the first population (Figure 1(a)) are 2.252 and 27.728, respectively. Table 1 is the average bias calculated based on the 5,000 samples. It appears that all the ratio estimators have the bias converging to zero, and it seems that the bias of the improved ratio estimators of $\hat{\mu}_{r,ht(s)}$ and $\hat{\mu}_{r,ht(ms)}$ converges to zero faster than others.

n	μ_y	$\hat{\mu}_r$	$\hat{\mu}_{r,hh}$	$\hat{\mu}_{r,hh(s)}$	$\hat{\mu}_{r,hh(ms)}$	$\hat{\mu}_{r,ht}$	$\hat{\mu}_{r,ht(s)}$	$\hat{\mu}_{r,ht(ms)}$
5	2.252	-0.412	-0.401	-0.403	-0.402	-0.401	-0.403	-0.403
10	2.252	-0.024	-0.021	-0.026	-0.024	-0.021	-0.025	-0.025
20	2.252	0.002	0.008	0.004	0.007	0.009	0.006	0.005
30	2.252	-0.001	-0.001	-0.006	-0.004	0.001	-0.004	-0.004
50	2.252	0.004	0.000	-0.002	0.001	0.002	0.001	0.000
100	2.252	0.003	-0.001	-0.002	-0.001	0.000	0.000	0.000
150	2.252	0.001	-0.001	-0.002	0.000	0.001	0.000	0.000
200	2.252	0.001	-0.001	-0.001	0.000	0.000	0.000	0.000

Table 1 Bias of various ratio estimators under the population in Figure 1 (a)



(a) A realized population of $\theta = 4, \theta_1 = \theta_2 = 200, \mu_y = 2.252$ and $\sigma_y^2 = 27.728$.



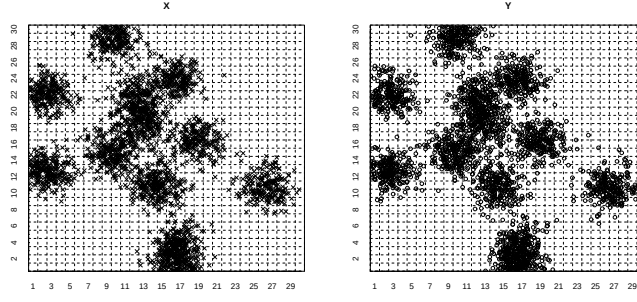
(b) E.R.E.

Fig. 1 The first population and E.R.E. of $\hat{\mu}_{r,(\dots)}$ with respect to the conventional $\hat{\mu}_r$.

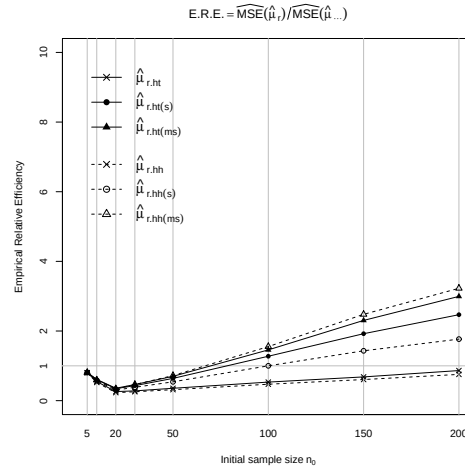
The simulation results summarized in Figure 1 (c) is the E.R.E. of the six ratio estimators of A.C.S to $\hat{\mu}_r$. The improved ratio estimators $\hat{\mu}_{r,(\dots)}$ ($\circ, \bullet, \triangle$, and $\blacktriangle$) are usually considerably better than their original versions $\hat{\mu}_{r,(\dots)}$ ($\times$) proposed in Dryver and Chao [4] under the same n_0 . While making use of sufficient statistic d^+ , the improved ratio estimator based on Horvitz-Thompson estimation ($\hat{\mu}_{r,ht(s)}, \bullet$) is better than the one based on Hansen-Hurwitz estimation ($\hat{\mu}_{r,hh(s)}, \circ$). The performances of the improved ratio estimator based on the minimal sufficient statistic d_R , $\hat{\mu}_{r,ht(ms)}$ ($\blacktriangle$) and $\hat{\mu}_{r,hh(ms)}$ ($\triangle$) are similar to each other and slightly better than $\hat{\mu}_{r,ht(s)}$. All of the proposed improved ratio estimators $\hat{\mu}_{r,(\dots)}$ are better than the conventional $\hat{\mu}_r$ under comparable final sample sizes when n_0 is greater than about 10, whereas the original versions $\hat{\mu}_{r,(\dots)}$ require larger n_0 be better than μ_r .

n	μ_y	$\hat{\mu}_r$	$\hat{\mu}_{r,hh}$	$\hat{\mu}_{r,hh(s)}$	$\hat{\mu}_{r,hh(ms)}$	$\hat{\mu}_{r,ht}$	$\hat{\mu}_{r,ht(s)}$	$\hat{\mu}_{r,ht(ms)}$
5	3.638	-0.104	-0.042	-0.048	-0.045	-0.039	-0.045	-0.048
10	3.638	-0.006	0.022	0.013	0.018	0.024	0.016	0.014
20	3.638	0.001	0.003	-0.004	0.002	0.007	0.001	-0.002
30	3.638	-0.001	-0.005	-0.008	-0.001	0.001	-0.002	-0.003
50	3.638	0.002	0.001	-0.001	0.002	0.004	0.002	0.001
100	3.638	0.002	-0.001	-0.002	0.001	0.002	0.001	0.001
150	3.638	0.002	-0.001	-0.002	-0.001	0.001	0.000	-0.001
200	3.638	0.000	0.001	0.000	0.001	0.002	0.001	0.000

Table 2 Bias of various ratio estimators under the population in Figure 2 (a)



(a) A realized population of $\theta = 10, \theta_1 = \theta_2 = 150, \mu_y = 3.636$ and $\sigma_y^2 = 29.577$



(b) E.R.E.

Fig. 2 The second population and E.R.E. of $\hat{\mu}_{r,(\cdot)}$ with respect to the conventional $\hat{\mu}_r$.

The second population (Figure 2(a)) has population mean 3.638 and variance 29.577. Please notice that it is a population in which ACS might be an appropriate design. The bias of all the $\hat{\mu}_{r,(\cdot)}$ converge to zero as n_0 increases (Table 2). Despite the similar pattern as seen in Figure 1 among the six ratio estimators under ACS, still there are other phenomenon different from the results in Figure 1. In fact, clearly it will not be as advantageous as the previous case to apply ACS since the population could no longer be regarded as a rare one as Figure 2(a) shows. For this population, $\hat{\mu}_{r,hh}$ and $\hat{\mu}_{r,ht}$ perform worse than the conventional $\hat{\mu}_r$. However, the improved ratio estimators $\hat{\mu}_{r,(\cdot)}$ can perform significantly better than $\hat{\mu}_r$, and they can be better than $\hat{\mu}_r$ under moderate or large n_0 . For example, the improved ratio estimators based on the minimal sufficient statistic d_R ($\hat{\mu}_{r,(ms)}$) and $\hat{\mu}_{r,ht(s)}$ are better than $\hat{\mu}_r$ when n_0 is greater than about 70, and $\hat{\mu}_{r,hh(s)}$ is better than $\hat{\mu}_r$ when $n_0 > 100$.

5 Discussion

Rao-Blackwellization is a well-known method for constructing better estimators for population quantity of interest. In ACS different initial samples can yield the same final sample, but do not always yield the same estimate for the estimators $\hat{\mu}_{hh}$, $\hat{\mu}_{ht}$, $\hat{\mu}_{r,hh}$, and $\hat{\mu}_{r,ht}$. Conditioning on the minimal sufficient statistic this source of sampling variability from ACS is eliminated. Conditioning on a sufficient statistic that

is not the minimal sufficient statistic can eliminate some to most of this source of sampling variability. Simplified analytical forms of Rao-Blackwellized estimators based on a sufficient statistic and the minimal sufficient statistic have been described in Dryver and Thompson [5] and Salehi [9] respectively for the univariate case.

In principle, the Rao-Blackwellized ratio estimation under ACS (Dryver and Chao [4]) can be calculated by taking all the possible combinations into account (e.g. Thompson [11]). The calculation is, however, usually impossible to carry out in practice. The analytical forms of the ratio estimator under ACS are currently unavailable. In this article we proposed various improved ratio estimators using the Rao-Blackwellized estimators proposed by Dryver and Thompson [5] and Salehi [9]. Though not the real Rao-Blackwellized versions, they are able to provide considerably better estimation results than the original ACS ratio estimators.

The ratio of the Rao-Blackwellized estimators given the minimal sufficient statistic yields the same estimate for the same final sample just as the Rao-Blackwellized ratio estimators conditioning on the minimal sufficient statistic would. In this sense although the approaches are different they conceptually achieve the same goal of eliminating the sampling variability arising from different initial samples that yield the same final sample. For this reason we believe the approach proposed in this article captures the majority of the benefits, if not all, that the Rao-Blackwellized ratio estimators based on the minimal sufficient statistic would produce. As a conclusion, it can be concluded that the proposed ratio estimators are preferable alternatives compared to the original versions.

In addition, ACS is known to be able to provide better estimation results than the comparable sampling designs when the population is a rare and/or cluster one. However, such information might not be clear before the survey to be conducted, and ACS can be an inappropriate choice in the perspective of statistical estimation when the population is not a rare one. The proposed ratio estimators also provide the investigators a chance to take advantage of other practical benefit of ACS, such as the higher sample yield, with comparable or better estimation result under such populations.

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