

Testing for Cointegration with Breaks in Dependent Panels

Francesca Di Iorio and Stefano Fachin

Abstract Allowing for structural breaks in the coefficients is a natural generalisation for any test of the existence of a relationship between a set of variables. Proposed test for cointegration with breaks which, however, tends to have rather poor power in small or moderate samples. In this paper we tackle the problem extending our procedure for cointegration testing in dependent panels based upon Residual-based Stationary Bootstrap. Simulation results suggest that the proposed panel cointegration test, full robust to general forms of dependence, has good size and power properties.

Key words: Time series, Co-breaking, Regression Trees.

1 Introduction

Allowing for structural breaks in the coefficients is a natural generalisation for any test of the existence of a relationship between a set of variables. In this vein Gregory and Hansen (1996) proposed a test for cointegration with breaks which, however, tends to have rather poor power in small or moderate samples. A further natural step forward is thus to construct a panel cointegration test with breaks, which can be expected to grant considerable power gains over pure time series procedures. Unfortunately, the empirical appeal of the currently available panel cointegration test with breaks is severely limited by their restrictive assumptions on the dependence structure of the panel: *e.g.* the tests by Westerlund (2006a,b) assume full cross-section independence, and both Banerjee and Carrion-i-Silvestre (2006) and Westerlund and Edgerton (2008) allow only for special types common factors. The former assume

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the common factors of the right-hand side variables to be orthogonal to those of the dependent variable, while the latter allow for common factors in the cointegrating residuals only. We must thus conclude that a panel cointegration test with breaks and full cross-section dependence is not available yet. In this paper we tackle the problem extending to the case of unknown breaks the procedure put forth in Di Iorio and Fachin (2008), hereafter DIF, who report promising results for a panel cointegration test with a break at a known date. As we will see, the proposed procedure, based upon Parker, Paparoditis and Politis's (2006) Residual-based Stationary Bootstrap (RSB) can account for fully general forms of dependence and delivers satisfactory small sample size and power properties. It must be stressed that since the interest here is on small sample testing and more general dependence structures the proposed procedure is seen as complementary, rather than alternative, to those based on factor methods; no comparison will be therefore be carried out.

We shall now first (section 2) introduce the set-up and outline the testing procedure, then (section 3) present the design and results of a Monte Carlo experiment. Some conclusions are finally drawn in section 4.

2 Bootstrap Panel Cointegration Testing with Breaks: Set-up

Let us consider for simplicity a standard bivariate panel cointegration set-up, with the variables of interest, denoted as usual by X and Y , observed over N units and T time periods. In each unit X and Y are linked by a linear, not necessarily cointegrating, relationship with a break in period t_i^b :

$$y_{it} = \begin{cases} \mu_{0i} + \beta_0 x_{it} + \varepsilon_{it}, & t \leq t_i^b \\ \mu_{1i} + \beta_1 x_{it} + \varepsilon_{it}, & t > t_i^b \end{cases} \quad (1)$$

with $i = 1, \dots, N$, $t = 1, \dots, T$. A panel cointegration test allowing for breaks may be defined very simply, following Pedroni's (1999) group mean test approach, as a summary statistic of the cointegration statistics with break computed for the individual units¹. This problem has been first tackled by Gregory and Hansen (1996), who proposed to compute a no-cointegration statistic $\theta(t_i^b)$, on the residuals of model (1) for all possible break points t_i^b (where $t_i^b \in [\delta T, (1 - \delta)T]$ and the trimming factor δ is chosen to ensure computational stability, with 0.15 or 0.20 popular choices) and, assuming the rejection region is the left tail (as in *ADF* and *Z* tests), take the minimum. The null hypothesis of the procedure is thus no cointegration, and the alternative hypothesis cointegration, with breaks allowed to be present in the linear relationship (1) in both cases. The break point is thus implicitly estimated as

$$\hat{t}_i^b = \underset{t_i^b \in [\delta T, (1 - \delta)T]}{\operatorname{argmin}} (\theta(t_i^b)). \quad (2)$$

¹ For simplicity we will refer to a summary statistic of the individual cointegration tests as a "panel test", although in Pedroni's terminology this term is reserved for tests obtained imposing an homogeneity assumption.

Another option, favoured by Westerlund and Edgerton (2008) and which we will follow, is to estimate the breakpoint on the basis of a least square criterion (which for stationary variables is consistent even under multiple breaks):

$$\hat{t}_i^b = \underset{t_i^b \in [\delta T, (1-\delta)T]}{\operatorname{argmin}} \left(\sum_{t=1}^T \varepsilon_{it}^2(t_i^b) \right). \quad (3)$$

where the notation $\varepsilon_{it}(t_i^b)$ for the residuals emphasises their dependence on the breakpoint used in the estimation of model (1). For each unit the set of residuals $\{\varepsilon_{it}(\hat{t}_i^b)\}_{t=1}^T$ is then by definition optimal in a least square sense, and can be used to compute a no-cointegration test with break. In fact, consider:

$$\varepsilon_{it} = \rho_i \varepsilon_{it-1} + v_{it}. \quad (4)$$

The hypothesis of no cointegration is equivalent to $H_0: \rho_i = 1$, and that of no panel cointegration as the same hypothesis for mean or median of this individual statistics, with the latter reflecting more closely the usual definition of the panel null hypothesis (no cointegration in the majority of the units; for a detailed discussion see DIF). Two important remarks are in order here. First, (4) is *not* a model of the cointegrating residuals; it only defines a parameter expressing the null hypothesis of interest. To evaluate the impact of studentising we shall use both the popular ADF statistic (ADF_i) and a coefficient statistic ($\tau_i = (\rho_i - 1)T$), used *e.g.*, by Palm, Smeekes and Urbain (2008). Second, the v_{it} 's are always stationary, either H_0 holds or not: they can thus be resampled via the stationary bootstrap. A bootstrap testing algorithm along the lines put forth in Parker *et al.* (2006) and already exploited in DIF, may then proceed as follows:

1. Estimate the breaking cointegrating model (1), with $\hat{t}_i^b$ given by (3), on the dataset $\{y_{it}, x_{it}\}$, $i = 1, \dots, N$, obtaining for each unit estimates of the coefficients $(\hat{\mu}_{ji}, \hat{\beta}_{ji}, j = 0, 1)$ and of the optimal cointegrating residuals $\{\hat{\varepsilon}_{it}\}$;
2. Compute the N individual no cointegration statistics $\hat{\theta}_i$, on the basis of the the optimal cointegrating residuals $\{\hat{\varepsilon}_{it}(\hat{t}_i^b)\}$;
3. Compute the Group statistics $\hat{\theta}_{mean} = N^{-1} \sum_{i=1}^N \hat{\theta}_i$, $\hat{\theta}_{median} = \operatorname{median}(\hat{\theta}_1 \dots \hat{\theta}_N)$;
4. Compute $\hat{v}_{it} = \hat{\varepsilon}_{it} - \hat{\rho}_i \hat{\varepsilon}_{it-1}$, where $\{\hat{\varepsilon}_{it}\}$ are the estimated residuals of the cointegrating relationship, and $\hat{\rho}_i$ is a consistent estimate estimate of ρ_i ;
5. Resample the matrix $[\hat{v}_1 \dots \hat{v}_N]$, where $\hat{v}_i = [\hat{v}_{i1} \dots \hat{v}_{iT}]'$ via the stationary bootstrap, obtaining $\{v_{it}^*\}$ (i.e. we resample all units simultaneously to preserve cross-dependence);
6. Cumulate $\{v_{it}^*\}$ obtaining pseudoresiduals $\{\varepsilon_{it}^*\}$ obeying the null hypothesis of no cointegration;
7. For each unit i build the pseudodata under the null hypothesis of no cointegration and break in $\hat{t}_i^b$:

$$y_{it}^* = \begin{cases} \hat{\mu}_{0i} + \hat{\beta}_0 x_{it} + \varepsilon_{it}^*, & t \leq \hat{t}_i^b \\ \hat{\mu}_{1i} + \hat{\beta}_1 x_{it} + \varepsilon_{it}^*, & t > \hat{t}_i^b \end{cases} \quad (5)$$

8. Using the datasets $\{y_{it}^*, x_{it}\}$ estimate the breaking cointegrating model (1) with the re-estimated breakpoints $\hat{t}_i^{b*}$ given by (3), obtaining sets of estimates of the optimal cointegrating residuals $\{\hat{\varepsilon}_{it}^*\}$;
9. Compute the individual bootstrap no cointegration statistics θ_i^* ;
10. Compute the bootstrap Group statistics θ_h^* ($h = \text{mean}, \text{median}$);
11. Repeat 5-11 B times;
12. Compute the bootstrap significance level of the statistics: $p(\theta)^* = \text{prop}(\theta_h^* < \theta_h)$.

Remark 1: the alternative hypothesis of this procedure is determined by the summary statistic. The median implies H_1 : "cointegration in the majority of the units", whereas the mean H_1 : "cointegration in the majority of the units *or* strong cointegration in a smaller number of units". Hence, the median is arguably a choice better reflecting the aim of the test.

Remark 2: Although exploratory simulations showed the results to be quite robust to the choice of block length, in principle this is a critical point of the algorithm. While we plan future investigations, here for computational convenience we fixed the block length at $T/10$, a simple choice which delivered good results, and at $1.75\sqrt[3]{T}$ as in Palm, Urbain and Smeekes (2008). This rule yields block sizes respectively slightly larger than $T/10$ for rather small sample sizes (e.g., 6 for $T = 40$), smaller for large sample sizes (e.g., 10 for $T = 160$). For moderate sample sizes the two rules suggest approximately the same block sizes. Since we will not use optimal block sizes we will somehow underrate the properties of the proposed test.

3 Monte Carlo Experiment

3.1 Design

We will base our simulations on a Data Generation Process (DGP) which is essentially a generalisation to the case of dependent panels of the classical bivariate DGP adopted by, e.g., Engle and Granger (1987) and Gonzalo (1994). It is very similar to that considered by Kao (1999), and it has been recently adopted by Gengenbach *et al.* (2006) and Di Iorio and Fachin (2008). As in the previous discussion, we assume two variables linked by a linear, not necessarily cointegrating, relationship with a break in period t_i^b :

$$y_{it} = \begin{cases} \mu_{0i} + \beta_0 x_{it} + \varepsilon_{it}^y, & t \leq t_i^b \\ \mu_{1i} + \beta_1 x_{it} + \varepsilon_{it}^y, & t > t_i^b \end{cases} \quad (6)$$

$$\varepsilon_{it}^y = \rho_i \varepsilon_{it-1}^y + e_{it}^y, \quad e_{it}^y \sim N(0, \sigma_{iy}^2) \quad (7)$$

where $i = 1, \dots, N$, $t = 1, \dots, T$. When X_i and Y_i are not cointegrated $\rho_i = 1$, while $|\rho_i| < 1$ when instead they are; in the power simulations ρ_i will be generated as $Uniform(0.6, 0.8)$ across units to mimick a generally rather slow adjustment to equilibrium. To ensure some heterogeneity across units $\sigma_{iy}^2 \sim Uniform(0.5, 1.5)$,

while with no loss of generality $\mu_{0i} = \beta_{0i} = 1 \forall i$; as in Westerlund and Edgerton (2008) we set both constant and slope to 5 after the break in all units. This is admittedly an extremely large break, of virtually no empirical relevance. However, setting the break to such a large value will permit us to evaluate the properties of the procedure independently on modelling difficulties.

Long-run growth of X is assumed to be driven by a non-stationary factor common across units (F_1), with short-run deviations caused by a second stationary common factor (F_2) and by an idiosyncratic stationary noise (ε_{it}^x):

$$x_{it} = \gamma_1 F_{1t} + \gamma_2 F_{2t} + \varepsilon_{it}^x \quad (8)$$

Following Pesaran (2007) the factor loadings are chosen so to ensure substantial cross-correlation in the X 's: $\gamma_i \sim \text{Uniform}(-1, 3) \forall i$. The common factors are generated as follows:

$$\begin{bmatrix} F_{1t} \\ F_{2t} \end{bmatrix} = \begin{bmatrix} F_{1t-1} \\ 0.4F_{2t-1} \end{bmatrix} + \begin{bmatrix} f_{1t} \\ f_{2t} \end{bmatrix} \quad (9)$$

where, as in Gengenbach *et al.* (2006), both the common and idiosyncratic shocks are assumed to have a MA(1) structure:

$$\begin{bmatrix} f_{1t} \\ f_{2t} \end{bmatrix} = \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix} + \begin{bmatrix} \vartheta_1 & 0 \\ 0 & \vartheta_2 \end{bmatrix} \begin{bmatrix} \eta_{1t-1} \\ \eta_{2t-1} \end{bmatrix} \quad (10)$$

$$\varepsilon_{it}^x = e_{it}^x + \varphi e_{it-1}^x, \quad (11)$$

where $\eta_{it} \sim N(0, 1)$, $i = 1, 2$, and $e_{it}^x \sim N(0, \sigma_{ix}^2)$, with $\sigma_{ix}^2 \sim \text{Uniform}(1, 1.4)$. Both φ and the ϑ 's are generated as $\text{Uniform}(0.5, 0.7)$. The simulation framework outlined above is very complex, and the tests to be evaluated computationally demanding (this issue is discussed in more detail below). Hence, rather than aiming at the unfeasible task of a complete design we will define as a base case an empirically relevant set-up and then explore a few interesting variations. Let us first discuss the design parameters common to all experiments. As standard in the literature we take the model as correctly specified. Given that in some of our simulations the time sample is quite small to ensure computational stability we choose a 20% trimming coefficient.

1. *Base case*: $T = 40$, $N = 5, 10, 20, 40$; break date Uniform over units in $[0.5T \pm 3] = [17, 23]$. The time span, medium in terms of annual data, is small at a quarterly frequency and smaller than those generally considered in the studies on the other cointegration tests with breaks available in the literature. The breaks are distributed over six periods centred in the middle of the time sample, with the testing procedure searching over the interval $[8, 32]$.
2. *Medium time sample*: $T = 80$, $N = 5, 10, 20$; break date Uniform over units in $[0.5T \pm 3] = [37, 43]$. The time span is now large in terms of annual data, but pretty common for quarterly data, so to make it relevant for actual empirical applications. To have results from this experiment closely comparable to those from the Base case, the sample is extended at both ends; hence, the breaks are

distributed over the same sets of periods centred in the middle of the time sample, with the procedure searching over the interval $[16, 64]$. Finally, since the interest here is on the behaviour of the test when the time dimension grows for computational convenience we limited the experiments to a most 20 units.

3. *Large time sample:* $T = 160, N = 5, 10$; break date Uniform over units in $[0.5T \pm 3] = [77, 83]$. The time span is long in terms of annual data, but medium with a quarterly frequency, so that it is still relevant for actual empirical applications. The sample is extended at both ends as in the previous case, and the search is over the interval $[32, 128]$. In this case also for computational convenience we consider only small cross-section sample sizes.

The last issue to be discussed is the number the number of Monte Carlo replications. This is chosen trying to strike a balance between the contrasting requirements of precision in the results and control of the cost and time scale of the experiment. Here this balance is particularly difficult to achieve because of the combined effects of the the panel structure of the data and the recursive nature of the statistics evaluated: the number of loops executed is the product of bootstrap redrawings, units, periods included in the searching interval, and number of Monte Carlo replications. With 500 bootstrap redrawings, 40 units and search over 28 periods, as in the Base Case, the product of first three terms is equal to 560.000. Fixing the Monte Carlo replications to 1000 will thus require the execution of over half a billion loops for each experiment, with, *e.g.*, for a rejection rate $p = 5\%$ an approximate confidence interval $p \pm 2\sqrt{p(1-p)/1000}$ equal to $[3.6\%, 6.4\%]$. Reducing the length of the interval even marginally to $[4.0\%, 6.0\%]$ requires a disproportionate effort, as the number of replications and hence that of loops would double. We thus decided that 1000 replications is a reasonable choice.

3.2 Results

The results are reported in Tables 1-5 below, and rapidly summarised. We denote the bootstrap test on the mean/median across units of the no cointegration statistics as *Boot-Mean* and *Boot-Median*. First of all, there seem to be very little differences between the performances of the group mean and median tests and, quite remarkably, block sizes considered. Hence, our comments can be expressed in general terms. Second, with a small time sample ($T = 40$) the Type I errors are somehow smaller than nominal sizes, but converge rapidly to the latter for $T = 80$. Finally, power can be disappointing for T and N both small, but increases rapidly with T and, most importantly, N . With the DGP used in our simulations the rejection rate of the false null of no cointegration is 100% already in almost all cases for $T = 80$ and always for $T = 160$ (hence, a separate table for this case is redundant). Clearly, these findings are conditional to speed of adjustment and signal-noise ratio. In systems with slower adjustment and more noise the power performances will not be as satisfactory, but this is not an issue: the key message is that with a small time sample a panel procedure may grant considerable power gains, while ensuring good size control.

4 Conclusions

Building upon Di Iorio and Fachin (2008), in this paper we propose to solve the problem of testing for the existence of a panel cointegrating relationships with breaks using the bootstrap; simulation results suggest that the proposed panel testing procedures may improve considerably on the small sample power performances of pure time series Gregory and Hansen (1996) tests. The good small sample performances obtained suggest it as a potentially useful addition to the toolbox for non-stationary panel analysis, complementary to asymptotic factor methods procedures (Banerjee and Carrion-i-Silvestre, 2006, Westerlund and Edgerton, 2008). Clearly, much methodological and empirical work is still needed. With respect to the latter, the natural next step forward is to test if the breaks actually took place; this is the subject of Di Iorio and Fachin (2007). On the former, just to mention a few: data-driven choice of the block size, multivariate DGP's and models, partially cointegrated panels.

Table 1: $T = 40$, N from 5 to 40 - Size

		N							
		5	10	20	40	5	10	20	40
Block	α	<i>Boot-Mean</i>				<i>Boot-Median</i>			
$0.10T = 4$	0.5	0.03	0.03	0.02	0.01	0.04	0.03	0.02	0.01
	0.10	0.07	0.07	0.05	0.03	0.08	0.06	0.06	0.05
$1.75\sqrt[3]{T} = 6$	0.05	0.03	0.02	0.01	0.00	0.03	0.02	0.01	0.01
	0.10	0.07	0.05	0.04	0.03	0.07	0.04	0.04	0.03

DGP: eqs. (6)-(11),

$t_i^b \sim \text{Uniform}(0.5T \pm 3)$, search interval: $[0.2T, 0.8T]$;

Table 2: $T = 80$, N from 5 to 20 - Size

		N					
		5	10	20	5	10	20
α		<i>Boot-Mean</i>			<i>Boot-Median</i>		
0.05	0.05	0.04	0.04	0.05	0.04	0.04	
0.10	0.10	0.09	0.09	0.10	0.09	0.09	

DGP: eqs. (6)-(11),

Block size: $8 \simeq 0.10T \simeq 1.75\sqrt[3]{T}$

Table 3: $T = 160$, $N = 5, 10$ - Size

		N			
		5	10	5	10
Block	α	<i>Boot-Mean</i>		<i>Boot-Median</i>	
$0.10T = 16$	0.05	0.04	0.02	0.05	0.05
	0.10	0.07	0.07	0.11	0.10
$1.75\sqrt[3]{T} = 10$	0.05	0.04	0.02	0.06	0.05
	0.10	0.07	0.06	0.10	0.10

DGP: eqs. (6)-(11),

Table 4: $T = 40$, N from 5 to 40, Power

		N							
		5	10	20	40	5	10	20	40
Block	α	<i>Boot-Mean</i>				<i>Boot-Median</i>			
0.10 $T = 4$	0.05	0.65	0.89	0.98	1.00	0.54	0.82	0.97	1.00
	0.10	0.81	0.95	0.99	1.00	0.72	0.93	0.99	1.00
1.75 $\sqrt[3]{T} = 6$	0.05	0.62	0.86	0.98	1.00	0.50	0.78	0.95	1.00
	0.10	0.79	0.95	0.99	1.00	0.69	0.91	0.99	1.00

DGP: eqs. (6)-(11), $\rho_i \sim \text{Uniform}(0.6, 0.8)$

Table 5: $T = 80$, N from 5 to 20 - Power

		N					
		5	10	20	5	10	20
α		<i>Boot-Mean</i>			<i>Boot-Median</i>		
0.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00

DGP: eqs. (6)-(11), $\rho_i \sim \text{Uniform}(0.6, 0.8)$;

Block size: $8 \simeq 0.10T \simeq 1.75\sqrt[3]{T}$;

5 References

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