

Simultaneous I-O tables balancing at current and constant prices

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Abstract: The balancing of a system of national accounts is normally carried out by balancing first the flows at current prices and then those at constant prices. However, this procedure in two distinct steps makes it impossible to “control” the consistency of the system of deflators. Therefore, a system of simultaneous balancing at current and constant prices is necessary to make it possible to obtain a consistent system of flows at current and constant prices, and of deflators. This paper introduces a simultaneous balancing method as an extension of the methodology described in Nicolardi (1998, 2000) for the simultaneous balancing at current and constant prices of a generic system of economic accounting. The application of the method to the Italian 2005 input-output table has yielded significant results in terms of balancing.

Keywords: National economic and social accounts; Statistical modelling; I-O tables; Simultaneous balancing method at current and constant prices.

1 Introduction

Most national accounting data is published by the National Institutes of Statistics at current prices and only for a small number of aggregates at constant prices. The reason behind this discrepancy is the considerable complexity of obtaining balanced accounting systems simultaneously at both constant and current prices. National accounts are, in fact, normally compiled and balanced first at current prices and then converted to constant prices using a suitable system of deflators. It is known that the accounting system at constant prices calculated in this way will be unbalanced and it is therefore necessary to balance it. This last balancing procedure, however, determines modifications in the value of the deflators and the latter may not reflect the true economic dynamic of the country to which the accounts refer. In the literature, it is possible to find some methods to solve the problem of balancing an accounting system at simultaneously current and constant prices. They are essentially based on the

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balancing method originally proposed by Stone, Champernowne and Meade (1942) and use an *ad hoc* transformation for variables, and the definition of a suitable matrix of variances and covariances to solve the nonlinearity of the accounting balancing system at both current and constant prices (Weale, 1988; Antonello, 1989; 1990). This work delineates a new method to simultaneously balance an accounting system at current and constant prices, which differs from the others in its approach to the problem. First, it is based on the method originally developed for balancing the Italian 1992 Symmetric Input-Output Table (SIOT, hereafter) (Nicolardi 1998; 2000) that had its foundation in Byron (1978). The method proposed by Nicolardi is more flexible than Byron's technique and allows the balancing of extremely large accounting systems. Second, the new method defined in this work utilizes the predictor-corrector method to solve the nonlinearity of the balancing system. In the end, it allows the direct control of the magnitude of the variations of the deflators, which the balancing procedure normally causes, while in the literature the problem is managed in a roundabout way. The paper is organized as follows. In Section 2 the methodology of balancing at current prices is presented. Section 3 introduces a method of simultaneous balancing at current and constant prices. Section 4 demonstrates an application of the method of simultaneous balancing. Section 5 presents some concluding remarks.

2 Methodological overview

Stone approached the balancing of an accounting system that includes differently reliable estimates of the aggregates as a problem of constrained estimates obtained as a weighted linear combination of the initial estimates of the same aggregates. By considering a vector $\mathbf{x}$ of s accounting data, it is possible to express a system of k accounting equations in the form $\mathbf{G}\mathbf{x} = \mathbf{h}$, where $\mathbf{G}$ is a $(k \times s)$ matrix of constraints and $\mathbf{h}$ is a vector of known values. In particular, when the accounting constraints are not satisfied, $\mathbf{G}\mathbf{x} \neq \mathbf{h}$ and the system is unbalanced. In this latter case, starting from an initial vector of unbalanced estimates of $\mathbf{x}$, say $\tilde{\mathbf{x}}$, Stone suggested to obtain a vector $\hat{\mathbf{x}}$ which satisfies the accounting constraints in $\mathbf{G}$ through a generalized constrained estimator defined as follows:

$$\hat{\mathbf{x}} = \tilde{\mathbf{x}} - \mathbf{V}\mathbf{G}'(\mathbf{G}\mathbf{V}\mathbf{G}')^{-1}(\mathbf{G}\tilde{\mathbf{x}} - \mathbf{h}) \quad (1)$$

where $\mathbf{V}$ is a prior estimate of the covariance matrix of $\hat{\mathbf{x}}$ and contains the *information* on the significance level of those estimates, and permits a reasoned distribution of the accounting residuals between them². Subsequently, Byron (1978) proposed an alternative approach to estimating $\hat{\mathbf{x}}$ that can be used to solve large econometric systems. He considered a constrained quadratic loss function defined as follows:

$$Z = \frac{1}{2}(\tilde{\mathbf{x}} - \hat{\mathbf{x}})' \mathbf{V}^{-1}(\tilde{\mathbf{x}} - \hat{\mathbf{x}}) + \boldsymbol{\lambda}'(\mathbf{G}\tilde{\mathbf{x}} - \mathbf{h}) \quad (2)$$

in which $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers. In (2), $\tilde{\mathbf{x}}$ has to be as close as possible, in a quadratic loss sense, to $\hat{\mathbf{x}}$ and satisfy, at the same time, the accounting restrictions. The first order conditions on (2) are:

² The matrix $\mathbf{V}$ is normally used in a diagonal form, where the null covariances of the estimates $\hat{\mathbf{x}}$ are assumed. In practical terms, the inefficiency of the estimates, which this simplification yields, is negligible in most cases. Also, the methodologies utilized in compiling the national accounts do not usually allow the estimation of the correlations between the estimates of different flows.

$$\tilde{\lambda} = (\mathbf{GVG}')^{-1}(\mathbf{G}\hat{\mathbf{x}} - \mathbf{h}) \quad (a); \quad \tilde{\mathbf{x}} = \hat{\mathbf{x}} - \mathbf{VG}\tilde{\lambda} \quad (b) \quad (3)$$

Although (3) is equivalent to (1), it allows the estimation problem to be solved under less restrictive conditions than (1), and, at the same time, the complexity of the estimation process to be reduced so that $\tilde{\mathbf{x}}$ can be worked out through a smaller linear equation system. Furthermore, Byron used the conjugate gradient method to estimate large accounting systems by applying (3). Recalling that (3a) is a system of linear equations and the matrix $(\mathbf{GVG}')$ is symmetric positive definite when the accounting system is exactly defined³, the same relation (3a) can be rewritten as follows:

$$(\mathbf{GVG}')\lambda = \mathbf{A}\lambda = \mathbf{q} \quad (4)$$

The solution to (4) can be obtained by using the following iterative procedure, i.e. the conjugate gradient algorithm:

$$\begin{aligned} \pi_0 &= \rho_0 = \theta - \mathbf{A}\lambda_0 \\ \alpha_i &= \rho_i' \rho_i / \pi_i' \mathbf{A} \pi_i \\ \lambda_{i+1} &= \lambda_i + \alpha_i \pi_i \\ \rho_{i+1} &= \rho_i - \alpha_i \mathbf{A} \pi_i \\ \beta_i &= \rho_{i+1}' \rho_{i+1} / \rho_i' \rho_i \\ \pi_{i+1} &= \rho_{i+1} + \beta_i \pi_i \end{aligned} \quad (5)$$

where π and ρ are the gradient-based direction vectors, λ_0 is a vector of initial values for λ , and i and $i+1$ refer to the iteration count. The iterative process is stopped when the condition $(\theta - \mathbf{A}\lambda_i) < \varepsilon$ is verified, with ε sufficiently small. Substituting λ_i for $\tilde{\lambda}$ in (3b) $\tilde{\mathbf{x}}$ is estimated. The convergence process can be accelerated by a suitable normalization of $(\mathbf{GVG}')$ (Byron, 1978). Also applying the balancing method proposed by Byron, the problem of calculating and managing $(\mathbf{GVG}')$ remains significant. In Nicolardi (1998, 2000) a method has been proposed of decomposing the accounting matrix into blocks that allows both a significant reduction of the quantity of data to memorize and the calculation of the product $(\mathbf{GVG}')$ without defining $\mathbf{G}$ element by element. The development of this methodology started from the observation that in (5) $(\mathbf{GVG}')$ appears in the second and fourth relations within the following product:

$$\mathbf{A}\pi = \mathbf{W}(\mathbf{GVG}')\pi \quad (6)$$

where $\mathbf{W}$ is the matrix of normalization proposed by Byron (1978). It can be seen in (6) that determining the product starting with the last pair of elements and proceeding to the first, the subsequent results are always formed by vectors, that is:

$$\mathbf{G}'_{(s \times k)} \pi_{(k)} = \Gamma_{(s)}; \quad \mathbf{V}_{(s \times s)} \Gamma_{(s)} = \Gamma'_{(s)}; \quad \mathbf{G}_{(s \times k)} \Gamma'_{(s)} = \Gamma''_{(s)}; \quad \mathbf{W}_{(k \times k)} \Gamma''_{(s)} = \Gamma'''_{(s)} \quad (7)$$

where the subscripts are the dimensions of the different matrices and vectors. As $\mathbf{V}$ and $\mathbf{W}$ are diagonal matrices and can be stored in vectorial form, it can easily be seen how the quantity of data to be memorized to obtain the product (6) is significantly reduced. In (7), however, the calculation of the product $(\mathbf{G}'\pi)$ remains a problem because of the significant dimensions that $\mathbf{G}$ can reach. To resolve this problem it was observed that the accounting matrices often have a block configuration, so that it was possible to reformulate the product $(\mathbf{G}'\pi)$ through a linear combination of suitable decomposition of π in blocks without having to determine $\mathbf{G}$ (Nicolardi 1998, 2000). A simple balancing scheme as defined below in (8a)⁴, where a_{ij} , b_{ij} and c_{ij} are respectively the ij -

³ In fact, the matrix $(\mathbf{GVG}')$ can not be positive definite mainly in two cases: when $\mathbf{G}$ has not full row rank, and this means that two or more constraint conditions are linearly dependent, and when the variances and covariances of the elements in a generic equation are all equal to 0. If the latter is not verified, it can be demonstrated that the matrix $(\mathbf{GVG}')$ is nonsingular when $\text{rank}(\mathbf{V}) \geq k$. In all cases, however, an appropriate reformulation of the system of equations can solve the problem.

⁴ In (8a) and (8b) the balancing equations 2-4 are already implicit in the first equation, but they are

th elements of the three generic $(n \times m)$ matrices of accounting data $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$, can easily be rewritten in a compact form (8b) as follows:

$$\left\{ \begin{array}{l} a_{ij} = b_{ij} + c_{ij} \quad \forall i, j \\ \sum_j a_{ij} = \sum_j b_{ij} + \sum_j c_{ij} \quad \forall i \\ \sum_i a_{ij} = \sum_i b_{ij} + \sum_i c_{ij} \quad \forall j \\ \sum_{i,j} a_{ij} = \sum_{i,j} b_{ij} + \sum_{i,j} c_{ij} \end{array} \right. \quad (a); \quad \left\{ \begin{array}{l} \mathbf{A} - \mathbf{B} - \mathbf{C} = \mathbf{0} \\ \mathbf{A} \mathbf{i}'_m - \mathbf{B} \mathbf{i}'_m - \mathbf{C} \mathbf{i}'_m = \mathbf{0} \\ \mathbf{i}_n \mathbf{A} - \mathbf{i}_n \mathbf{B} - \mathbf{i}_n \mathbf{C} = \mathbf{0} \\ \mathbf{i}_n \mathbf{A} \mathbf{i}'_m - \mathbf{i}_n \mathbf{B} \mathbf{i}'_m - \mathbf{i}_n \mathbf{C} \mathbf{i}'_m = \mathbf{0} \end{array} \right. \quad (b) \quad (8)$$

where $\mathbf{i}_n$ and $\mathbf{i}_m$ are vectors of ones of dimension n and m respectively. In (8b) only q matrices, vectors and scalars are formed by initial data, while the remainder come from their transformations, e.g. $\mathbf{A} \cdot \mathbf{i}'_m$. It is now possible to calculate the vectors $\mathbf{\Gamma}$ and $\mathbf{\Gamma}''$ in (7) without using explicitly $\mathbf{G}$ by means of the formulation (8b). In particular, in order to compute the product $\mathbf{G}'\mathbf{\pi}$ in the first of (7), $\mathbf{\pi}$ must be decomposed into k blocks $\mathcal{S}_i$ (matrices, vectors and scalars) equal to the number of macro-equations of scheme (8b). Each of the k blocks has the same dimensions as the data matrices, or their transformations, which enter each macro-equation. Through the application of the scheme (8b), $k = 4$ blocks with dimensions, respectively, $(n \times m)$, $(n \times 1)$, $(1 \times m)$ e (1×1) are obtained. The solution of the above product $\mathbf{G}'\mathbf{\pi}$ is given by the q blocks $\mathcal{R}_i$ that are calculated as a linear combination of the blocks $\mathcal{S}_j$, where j corresponds to the macro-equations in which each of the initial q matrices of scheme (8b), ordered according to the sequential position in the balancing scheme and without repetition, is present. For instance, when the scheme (8b) is used, $q = 3$ since the only matrices of initial data are $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$, and the ordered vector of initial matrices is $\mathbf{\zeta} = (\mathbf{A}, \mathbf{B}, \mathbf{C})$. If a $(q \times k)$ matrix $\mathbf{\Lambda}$ is specified, where each row i contains the identification numbers of the macro-equations where the i -th matrix in $\mathbf{\zeta}$ is present, the i -th block $\mathcal{R}_i$ can be worked out by using the first of (9) below. In that formula, $\mathbf{\Lambda}_i$ denotes the i -th row of the matrix $\mathbf{\Lambda}$ and the superscript el indicates that all the sums are to be computed element by element⁵. Each block $\mathcal{S}_j$ has to experience the same algebraic sign of the i -th matrix in $\mathbf{\zeta}$ in each k -th equation. The second and third of (9) show the matrix $\mathbf{\Lambda}$ and the formulae to calculate each block $\mathcal{R}_i$, as follows:

$$\mathcal{R}_i = \left(\sum_{j \in \Lambda_i} \mathcal{S}_j \right)^{el} \quad i = 1, \dots, q; \quad \mathbf{\Lambda} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}; \quad \left\{ \begin{array}{l} \mathcal{R}_1 = +\mathcal{S}_1 + \mathcal{S}_2 \mathbf{i}'_m + \mathbf{i}_n \mathcal{S}_3 + \mathcal{S}_4 \mathbf{i}_n \mathbf{i}'_m \\ \mathcal{R}_2 = -\mathcal{S}_1 - \mathcal{S}_2 \mathbf{i}'_m - \mathbf{i}_n \mathcal{S}_3 - \mathcal{S}_4 \mathbf{i}_n \mathbf{i}'_m \\ \mathcal{R}_3 = -\mathcal{S}_1 - \mathcal{S}_2 \mathbf{i}'_m - \mathbf{i}_n \mathcal{S}_3 - \mathcal{S}_4 \mathbf{i}_n \mathbf{i}'_m \end{array} \right. \quad (9)$$

The sequential vectorization of the q blocks yields $\mathbf{\Gamma}$. To determine $\mathbf{\Gamma}''$ in the third of (7), $\mathbf{\Gamma}'$ must be decomposed into q blocks $\mathcal{S}_i$ with the same dimensions as the i -th data matrix in $\mathbf{\zeta}$. The solution of the product $\mathbf{G}\mathbf{\Gamma}'$ is given by the k blocks $\mathcal{R}_i$ as resulting from the linear combination of the blocks $\mathcal{S}_j$ with the same position as the matrices $\mathcal{S}_j$ in each macro-equation of (8b). If a $(k \times q)$ matrix $\mathbf{\Lambda}$ is specified, where each row i contains the position of each j -th data matrix in $\mathbf{\zeta}$ in each j -th equation in (8b), the i -th block $\mathcal{R}_i$ can be worked out by using the first of (10) below. In that formula, $\mathbf{\Lambda}_i$ denotes the i -th row of the matrix $\mathbf{\Lambda}$, and the superscript tr indicates that the same transformation as the ij -th element in the scheme (8b) is to be applied to the block $\mathcal{S}_j$. Each block $\mathcal{S}_j$ has to experience the same algebraic sign of the j -th matrix in $\mathbf{\zeta}$ in each

normally used explicitly so as to be able to express the constraint conditions on the marginal distributions of the accounting matrices.

⁵ The following are defined: 1) the sum element by element of a matrix $\mathbf{A}$ and a column vector $\mathbf{b}$ the matrix $\mathbf{C} = \mathbf{A} + \mathbf{b} \mathbf{i}'$; 2) the sum element by element of a matrix $\mathbf{A}$ and a row vector $\mathbf{b}$ the matrix $\mathbf{C} = \mathbf{A} + \mathbf{i} \mathbf{b}$; 3) the sum element by element of a matrix $\mathbf{A}$ and a scalar b , the matrix $\mathbf{C} = \mathbf{A} + b (\mathbf{i} \mathbf{i}')$; 4) the sum element by element of a vector $\mathbf{a}$ and a scalar b the vector: $\mathbf{c} = \mathbf{a} + b \mathbf{i}$. The definition of the sum element by element of two matrices or two vectors are not given because they are obvious,

i -th equation. The second and third of (10) show the matrix $\mathbf{A}$ and the formulae to calculate each block $\mathfrak{R}_i$. The sequential vectorization of the k blocks yields $\mathbf{\Gamma}'$.

$$\mathfrak{R}_i = \left(\sum_{j \in \Lambda_i} \mathfrak{S}_j^r \right) \quad i = 1, \dots, k; \quad \mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}; \quad \begin{cases} \mathfrak{R}_1 = +\mathfrak{S}_1 - \mathfrak{S}_2 - \mathfrak{S}_3 \\ \mathfrak{R}_2 = +\mathfrak{S}_1 \mathbf{i}_m - \mathfrak{S}_2 \mathbf{i}_m - \mathfrak{S}_3 \mathbf{i}_m \\ \mathfrak{R}_3 = +\mathbf{i}'_n \mathfrak{S}_1 - \mathbf{i}'_n \mathfrak{S}_2 - \mathbf{i}'_n \mathfrak{S}_3 \\ \mathfrak{R}_4 = +\mathbf{i}'_n \mathfrak{S}_1 \mathbf{i}_m - \mathbf{i}'_n \mathfrak{S}_2 \mathbf{i}_m - \mathbf{i}'_n \mathfrak{S}_3 \mathbf{i}_m \end{cases} \quad (10)$$

3 A method for the simultaneous balancing of an accounting system at current and constant prices

The main problem of simultaneously balancing an accounting system at current and constant prices is related to the resolution of a nonlinear system of simultaneous equations. Let $\mathbf{A}^c$ indicate a generic matrix of accounting data at current prices, $\mathbf{A}^k$ the same matrix at constant prices, and $\mathbf{A}^i$ the corresponding deflator. It is now necessary to add to the above system (8b) a set of equations which permit the simultaneous balancing of all the matrices. Considering only the first row in (8b), the simultaneous system of equations is defined as follows:

$$\begin{aligned} \mathbf{A}^c - \mathbf{B}^c - \mathbf{C}^c &= \mathbf{0} \\ \mathbf{A}^k - \mathbf{B}^k - \mathbf{C}^k &= \mathbf{0} \\ (\mathbf{A}^c \cdot \mathbf{A}^i) - \mathbf{A}^k &= \mathbf{0} \\ (\mathbf{B}^c \cdot \mathbf{B}^i) - \mathbf{B}^k &= \mathbf{0} \\ (\mathbf{C}^c \cdot \mathbf{C}^i) - \mathbf{C}^k &= \mathbf{0} \end{aligned} \quad (11)$$

Clearly, the above system cannot be resolved with the method described in Section 2 because it is nonlinear. Given a general accounting system like (8), its simultaneous balancing at current and constant prices can be worked out by using a procedure based on the predictor-corrector method. For the sake of brevity, only the system (11) is considered and rewritten as follows:

$$\begin{aligned} \text{Step 1} \quad \{ \mathbf{A}^c - \mathbf{B}^c - \mathbf{C}^c = \mathbf{0}; \quad & \text{Step 2} \quad \begin{cases} \log(\mathbf{A}^c) + \log(\mathbf{A}^i) - \log(\mathbf{A}^k) = \mathbf{0} \\ \log(\mathbf{B}^c) + \log(\mathbf{B}^i) - \log(\mathbf{B}^k) = \mathbf{0} \\ \log(\mathbf{C}^c) + \log(\mathbf{C}^i) - \log(\mathbf{C}^k) = \mathbf{0} \end{cases} \quad \text{Step 3} \quad \{ \mathbf{A}^k - \mathbf{B}^k - \mathbf{C}^k = \mathbf{0} \end{aligned} \quad (12)$$

where each group of equations represents a linear system. The general solution to the problem, therefore, lies in applying a suitable iterative method of estimation for Steps 1-3 until a balanced accounting system is obtained, at both current and constant prices. At each step, the starting values are the balanced estimates worked out in the previous step. In the practical application of the iterative procedure, Step 2 must be utilized twice: the first time to obtain the estimates of the flows at constant prices to use in Step 3, the second to obtain the estimates of the flows at current prices to use in Step 1 of the next iteration. A worthwhile aspect of this method is its ability to control the variations induced in the price system by the balancing procedure through the definition of an suitable variance matrix for the deflators. Comparing (12) and (8b), it is clearly seen how the block balancing procedure described in Section 2 of this paper can be utilized to balance the accounting system at every single step of the simultaneous procedure.

4 An application to the Italian 2005 SIOT

The simultaneous balancing procedure at current and constant prices was tested on the Italian 2005 SIOT at basic prices⁶. This matrix, published by the Italian National Institute of statistics (ISTAT hereafter), is compiled with the total flows of 59 branches of production at current prices only and is, obviously, already balanced. The first step was therefore to obtain a non-balanced matrix by modifying the data with a random algorithm. The modifications obtained were held within a maximum limit of 5%. An *ad hoc* estimate of the 2005 SIOT with 2000 as price-base year was, then, worked out. For this, where they were available, the flows at constant prices of the national accounts were used, or the price indices of both input and output also published by ISTAT. Table 1 shows the 2005 SIOT at current and constant prices in an aggregated form, according to the A6 classification in 1995 ESA. The last column of the table reports the vector $\mathbf{Gx}$ of residuals.

Table 1: 2005 SIOT at current price (c) and price-base year 2000 (k) . Unbalanced flows. Basic prices. Billions of Euro.⁷

Branches		A1	A2	A3	A4	A5	A6	IUBP	FCE	GCF	EFOB	TUBP	Res.
A1	c	5.3	28.6	0.1	6.4	0.5	0.5	41.2	11.3	0.2	4.3	57.6	1.2
	k	5.0	27.1	0.1	6.0	0.4	0.5	39.9	10.0	0.2	3.9	53.2	-1.5
A2	c	8.2	442.0	49.1	112.6	22.3	28.6	663.6	220.3	98.7	265.3	1245.9	5.7
	k	7.4	401.6	43.6	102.3	19.5	25.4	605.0	193.0	87.2	238.8	1115.9	-33.8
A3	c	0.2	8.7	17.0	12.7	8.2	4.6	52.1	7.3	128.1	1.0	187.1	3.8
	k	0.2	7.5	14.6	11.4	6.9	3.9	43.7	6.4	113.1	0.9	160.4	3.9
A4	c	3.3	124.4	21.5	132.1	31.8	18.9	328.0	316.6	29.8	49.2	720.3	-6.0
	k	2.9	113.8	19.2	120.6	28.3	17.0	300.5	280.0	26.3	44.3	641.4	-9.0
A5	c	1.3	68.1	16.9	109.3	101.6	36.0	333.0	164.7	24.5	19.8	550.9	7.4
	k	1.1	60.5	14.7	96.4	86.7	31.2	284.3	146.1	21.6	17.8	472.6	11.8
A6	c	0.2	10.2	1.9	9.4	4.2	23.5	49.7	341.7	1.1	1.2	394.9	5.9
	k	0.2	8.9	1.7	8.3	3.5	20.2	42.0	301.3	1.0	1.1	341.5	8.3
ICBP	c	18.3	682.1	107.8	385.6	169.7	111.3	1499.8	1084.1	280.5	343.5	3209.4	-14.4
	k	16.6	629.1	94.7	345.6	142.4	97.3	1320.5	945.2	247.7	309.1	2818.1	-37.8
NT	c	0.5	10.0	1.6	10.2	6.5	9.5	38.1					
	k	0.5	9.0	1.4	8.9	5.4	8.2	33.1					
VABP	c	28.4	270.3	77.4	298.1	347.9	268.5	1314.7					
	k	29.8	248.2	61.1	263.0	292.5	226.3	1139.5					
ICIF	c	9.8	290.7	1.1	34.4	24.1	2.2	362.3					
	k	9.0	266.9	1.0	31.6	22.1	2.0	332.6					
TSBP	c	56.4	1240.2	183.3	726.3	543.6	388.9	3223.8					
	k	54.7	1149.6	156.5	650.4	460.8	333.2	2855.9					

The balancing procedure for the SIOT at current and constant prices was divided in 3 steps, as in (12), in each of which the block balancing method was used. The system of balancing equations was defined in a very analytical way, using a scheme similar to that described by Nicolardi (2000). For *Steps 1* and *3*, 7458 equations were utilized, with a vector $\hat{\mathbf{x}}$ of 11464 elements, while for *Step 2* the number of equations was 11464 with a vector $\hat{\mathbf{x}}$ of 34392 elements⁸. In order to ensure the convergence of the entire system, a very complex scheme was adopted for the determination of the estimate

⁶ All the data used in this work are taken from the online database of ISTAT.

⁷ **A1-A6** Branches of production; **ICBP** - Intermediate consumption at basic prices; **NT** - Net taxes; **VABP** Value added at basic prices; **ICIF** - Imports cif; **TSBP** - Total supply at basic prices; **IUBP** - Intermediate uses at basic prices; **FCE** - Final consumption expenditure; **GCF** - Gross capital formation; **EFOB** - Exports fob; **TUBP** - Total uses at basic prices.

⁸ The complete balancing scheme is available on request from the author.

variances of the different accounting flows. In broad terms, variable weights between 0 and 1 were attributed to the initial values and subsequently the variances were obtained through the product of each estimate for the respective weight (Nicolardi, 2000). Specifically, the different significance of the estimates obtained directly from the national accounts and of those obtained with indirect methods was taken into account in determining the variances of the values at constant prices and those of the deflators. It should also be recalled that as the starting vector of the estimates in every iteration is the final vector of the preceding one, it is necessary to recalculate the variance matrix at each iteration. In this work, the criterion adopted was to leave the matrices of initial weights unchanged and to recalculate the new variance matrices multiplying them by the new estimates. One of the critical points of the convergence process is certainly *Step 2*. Indeed, without any restriction on the partial convergence process of the step, there is a possibility that the system may not converge, or that it may converge very slowly. To avoid this, a general criterion of calibration for the variances of the different flows for *Step 2* only was adopted. Therefore, the variances of the data at current prices were significantly reduced, while those of the deflators underwent a lower reduction. The variances of the data at constant prices were left unaltered. The different calibration depicted above focuses much more on the balancing of the flows at current prices compared to that of flows at constant prices. These latter estimates, in fact, represent a “residual” output with respect to the calculation of flows at current prices and of the deflators. Moreover, as the variances have the same measurement unit as the flows, it was necessary to normalize them in *Step 2* so as to permit a distribution of the residuals among the non-balanced estimates according to a uniform criterion. To obtain a more rapid convergence of the method, the solution adopted was to reduce gradually the weights for the recalculation of the variance matrices of the flows at current prices at each iteration. A further problem in *Step 2* is related to the necessity of calculating the logarithms of the estimates in $\hat{x}$. The problem is due to the fact that there may be negative values in the SIOT (e.g. change in inventories), for which it is not possible to calculate the value of the logarithm. In this work, the criterion adopted is to consider the absolute value of those negative values and then invert the operation in the relative balancing equation. The condition $\max(|\mathbf{e}|) < 0.1$ was placed as a stop criterion of the iterative procedure for the accounting system at both current and constant prices in the same iteration. Table 2 reports the 2005 SIOT at constant prices.

5 Conclusions

In this paper, a balancing procedure for I-O tables has been introduced to permit their simultaneous balancing at current and constant prices. Comparison of the balanced and non-balanced 59 branch SIOT matrices shows that the simultaneous balancing procedure yields “reasonable” changes in the initial estimates from an economic and accounting point of view. In fact, the balancing system re-valued the flows at current prices by an average of $\pm 3\%$. The balancing process also yielded greater modifications to the deflators and to the flows at constant prices, as was to be expected given the indirect methods used for determining the initial estimates. Nevertheless, by analyzing in detail the variations yielded by the balancing procedure, it can be noted that the greatest relative variations involve the flows with negligible values, mainly in the matrix of intermediate consumption and the vector of the change in inventories. In all the other cases, the relative variation is less than 10%. Furthermore, the convergence of the balancing procedure was very rapid: 43 iterations were sufficient to obtain the

balancing of the accounting system. These results suggest that the procedure proposed in this paper is suitable to solve the still relevant problem of simultaneous balancing at current and constant prices of the national accounting systems. Further future developments could emerge mainly from the search for stronger conditions for the convergence of the iterative process, and also for more efficient criteria of recalculation and management of the variance matrices in the different steps of the procedure.

Table 2: 2005 SIOT at current price (c) and price base year 2000 (k). Balanced flows. Basic prices. Billions of Euro.

<i>Branches</i>		<i>A1</i>	<i>A2</i>	<i>A3</i>	<i>A4</i>	<i>A5</i>	<i>A6</i>	<i>IUBP</i>	<i>FCE</i>	<i>GCF</i>	<i>EFOB</i>	<i>UBP</i>
A1	<i>c</i>	5.3	28.7	0.1	6.4	0.5	0.5	41.3	11.4	0.2	4.3	57.3
	<i>k</i>	5.0	27.8	0.1	6.3	0.5	0.5	40.2	10.4	0.2	4.0	54.7
A2	<i>c</i>	8.3	444.7	49.2	114.0	22.5	28.7	667.5	222.0	98.9	266.3	1254.6
	<i>k</i>	7.3	408.4	44.5	104.8	20.0	26.2	611.3	197.5	90.2	240.8	1139.8
A3	<i>c</i>	0.2	8.8	17.3	13.0	8.4	4.7	52.4	7.5	125.2	1.0	186.1
	<i>k</i>	0.2	7.1	14.3	10.7	6.5	3.7	42.5	6.5	108.9	0.9	158.7
A4	<i>c</i>	3.3	124.3	21.3	133.0	32.0	18.9	332.8	316.7	29.4	49.6	728.6
	<i>k</i>	2.8	112.4	19.2	119.8	27.8	16.8	298.8	278.4	26.3	43.9	647.4
A5	<i>c</i>	1.3	68.9	17.0	111.3	102.9	36.5	338.0	167.8	24.1	19.8	549.7
	<i>k</i>	1.0	58.8	14.7	93.4	84.7	31.1	283.7	145.7	21.0	17.4	468.0
A6	<i>c</i>	0.2	10.3	1.9	9.5	4.3	23.4	49.6	342.5	1.1	1.3	394.5
	<i>k</i>	0.1	8.4	1.6	7.9	3.2	19.6	40.9	297.3	1.0	1.1	340.3
ICBP	<i>c</i>	18.5	685.8	106.7	387.1	170.6	112.8	1481.5	1067.9	279.0	342.2	3170.8
	<i>k</i>	16.4	622.9	94.4	342.9	142.7	98.0	1317.3	935.8	247.6	308.1	2808.8
NT	<i>c</i>	0.5	9.8	1.6	10.0	6.4	9.4	37.7				
	<i>k</i>	0.4	8.8	1.4	8.6	5.3	8.2	32.7				
VABP	<i>c</i>	28.4	270.2	76.7	297.3	348.9	270.2	1291.7				
	<i>k</i>	29.3	245.4	61.8	264.1	297.8	232.0	1130.4				
ICIF	<i>c</i>	9.8	288.8	1.0	34.1	23.8	2.2	359.8				
	<i>k</i>	8.5	262.8	1.0	31.9	22.2	2.1	328.4				
TSBP	<i>c</i>	57.3	1254.6	186.1	728.6	549.7	394.5	3170.8				
	<i>k</i>	54.7	1139.8	158.7	647.4	468.0	340.3	2808.8				

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