

# Empirical Mode Decomposition for Trend Extraction. Application to Electrical Data

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**Abstract** This paper presents a method for trend extraction from seasonal time series through the Empirical Mode Decomposition (EMD). Experimental comparison of trend extraction based on EMD and Hodrick Prescott filter are conducted. First results proved the eligibility of EMD trend extraction. Tunisian real peak load is used to illustrate the extraction of the intrinsic trend. Finally a remark about wavelets, a natural competitive method for EMD, reinforce the conclusion that that EMD trend extraction method does not require any tuning parameter or any wavelet choice thanks to its adaptive nature.

## 1 Introduction

Time series often contain many components such as seasonal and cyclical components, trends and irregularities, if we assume an additive decomposition model. However, even under this simplified form, trend extraction and seasonal adjustments are difficult tasks of time series analysis, due to the extreme variety of time series with their own time scales. Thereby, the trend has fuzzy general definition, despite its great practical importance. Nevertheless, it is considered to be a "smooth additive component that contain information about global change", see [1]. This adopted definition make trend extraction an ambiguous task since we can found many several candidates from a time series which match this definition. Therefore, trend extraction should be related to time scales. For example, it is of great interest to extract very smoothed trend components from time series for long- or medium-term load forecasting.

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Assuming additive decomposition model of time series, we will investigate in this paper the eligibility of trend extraction based on the Empirical Mode Decomposition (EMD), introduced by Huang et al. (see [3]). The EMD method is useful particularly to deal with possibly nonstationary and nonlinear which often characterize time series. The motivation to use the EMD is that it considers the signal as a superposition local sums of oscillatory components, extracted from upper and lower envelope, so-called Intrinsic Mode Functions (IMF). The IMFs are fully data-driven and local in time. This is important since it allows to identify various trends at different time scales. This method is easy to implement and does not use any predetermined transforms which depend on the choice of a particular theoretical structure. Furthermore, the EMD is an adaptive method which is entirely empirical and captures the characteristics in separate IMFs, explaining why it has been successfully applied in many engineering fields, see e.g. [2], [11].

The outline of the paper<sup>1</sup> is as follows. Section 2 recalls some facts about Empirical Mode Decomposition and sketches how it is a good candidate for knowledge extraction. Section 3 experiments EMD-based trend extraction method on simulated seasonal time series, and compares it to the moving average (MA) filtering and the widely used trend extraction method based on Hodrick-Prescott filter (see e.g. [6]). Then, Section 4 illustrates the method on real data by extracting trend component of the Tunisian peak load<sup>2</sup> from 2000 to 2006. This section includes a final remark about wavelets, a natural competitive method for EMD.

## 2 Empirical Mode Decomposition and intrinsic trend

### 2.1 Time scales and trend extraction of time series

Let us consider observed additive time series  $y = (y_1, y_2, \dots, y_{T_{Obs}})$  supposed to be of the form:

$$y_t = T_t + S_t + C_t + I_t \quad (1)$$

where the different components are trend ( $T$ ), seasonal components ( $S$ ), cycles ( $C$ ) and irregular term ( $I$ ) for error modeling. With respect to this usual form, we will not make distinction between seasonal and cyclical components, in order to make these components identifiable. So, the signal decomposition reduces to :

$$y_t = T_t + SC_t + I_t \quad (2)$$

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where  $SC$  represents seasonal and cyclical components. Several techniques have been traditionally used for time series components extraction and adjustment. As mentioned above, we will focus on trend extraction. Let us mention some the most frequently used approaches (see [1] for a recent review) for trend extraction such as local or global regressions, moving average filtering, X11, X12 and the Hodrick Prescott filter.

## 2.2 EMD residue as intrinsic trend

Empirical Mode Decomposition (EMD) has been introduced by Huang et al. [3], as an important alternative to traditional methods for analyzing time series such as wavelets or Fourier methods. The key idea of EMD is to locally decompose data  $y_t$  into oscillatory components so-called Intrinsic Mode Functions (IMF). The algorithm for the extraction of IMFs from a given time series  $y_t$  data is called sifting and it consists of the following steps:

- i Initialize the residue  $r_0(t) = y_t$ , set  $g_0(t) = r_{k-1}(t)$  and  $i = 1$ ; the index of IMF  $k = 1$
- ii Construct the lower minima  $Imin_{i-1}$  and the upper maxima  $Imax_{i-1}$  envelopes of the signal by the cubic spline method
- iii Calculate the mean values by averaging the upper envelope and the lower envelope. Set  $m_{i-1} = [Imax_{i-1} + Imin_{i-1}]/2$
- iv Subtract the mean from the original signal  
 $g_i = g_{i-1} - m_{i-1}$  and  $i = i + 1$ ,  
 and repeat steps (ii)-(iv) until  $g_i$  being an IMF (see below). If so, the  $k$ th IMF is given by  $IMF_k = g_i$
- v Update residue  $r_k(t) = r_{k-1}(n) - IMF_k$ . This residual component is treated as a new data and subjected to the process described above to calculate the next  $IMF_{k+1}$ .
- vi Repeat the steps above until the final residual component  $r(t)$  becomes a monotonic function.

It turns out that an IMF satisfy the two following properties. First: the upper and lower envelopes are symmetric and second: the number of zero-crossings and the number of extrema are equal or differ at most by one.

The advantage of this method is the fact that the oscillatory modes which are generated, are derived directly from the data without any reference to a predetermined dictionary of functions.

At the end of this process, the initial time series is decomposed into  $K$  IMF components and  $r$  is the final residue :

$$y_t = \sum_{k=1}^K IMF_k(t) + r(t) \quad (3)$$

Such a decomposition offers the opportunity to consider that  $r(t)$  as estimate of the trend of the data. Since, at the end of the algorithm, the number of extrema in the residue does not exceed 2. These results make EMD algorithm very suitable to extract trend. Note that end effect can affect the goodness fit of the trend extracted through EMD (see e.g. [7]). In this case, no physical meaning IMFs can be obtained and the exact trend can be reconstructed by aggregation of the residue and the last or the last two IMFs.

### 2.3 Trend definitions and EMD

In this section we present a short overview of previous studies dealing with trend extraction through EMD. It is important to note that there are only a few references, namely [9], [2] and [10], and that there is no consensus about how to define trend, since trend definitions are related to the data peculiarities and fields of application.

Flandrin et al. [2] have investigated the potentialities and limitations of EMD-based methods in detrending, relating the trend with the statistical properties of the IMFs. Indeed trend is defined as the sum of the IMFs having non-zero mean  $T_t = \sum_{k>D} IMF_k(t)$ . Application to heart-rate data illustrates its potential detrending usefulness.

Another definition is given in [9], relating trend to time scales and  $T_t$  is supposed to be the trend of  $y_t$  on time scale  $T$  if  $\exists (t_1, t_2), (t_2 - t_1) > T$  such that  $(T_{t_2} - T_{t_1})(y_{t_2} - y_{t_1}) \geq 0$ . A short and partial comparison between EMD and a specific Moving Average method is made provided using Stock P&G time series.

Finally, let us mention that in [11], Zhou et al. have proposed an algorithm for removing trends from power-system oscillation data based on a slightly modified EMD. This ad-hoc adaptation is developed especially for highly oscillatory data.

In our case, trend definition and extraction are related to time scale. We investigate the performance of EMD-based approach for extraction classical long-term trend.

## 3 Simulated examples

### 3.1 Simulated seasonal series

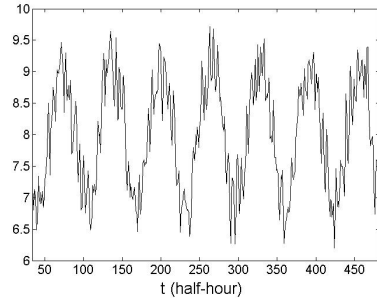
The empirical EMD characteristic make difficult to quantify the EMD trend performance method analytically. For this, we will investigate its performance through experimental studies.

We consider elementary sinusoidal modeling for simulated daily power pattern ( $X_t$ ), supposed to be sampled every 22 minutes (see Figure 1.a). To examine trend extraction issue through EMD, a modified version ( $y_t$ ) of this simulated time series

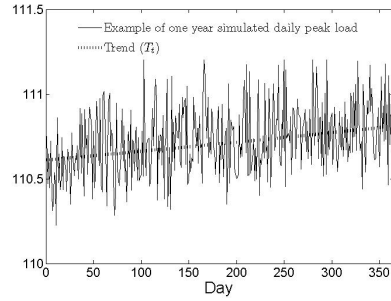
( $X_t$ ) is obtained by adding classical trends (linear and exponential), even if they are unrealistic. The complete model is given by the following equations:

$$\begin{cases} y_t = X_t + T_t. \\ X_t = \beta_0 + \beta_1 m_1(t) + \beta_2 m_2(t) + \varepsilon(t) \\ m_1(t) = \cos(\frac{2\pi t}{64}) + \sin(\frac{2\pi t}{64}) \\ m_2(t) = \cos(\frac{2\pi t}{6}) + \sin(\frac{2\pi t}{6}) \\ \varepsilon(t) = v(t) + \theta v(t-1) \quad v(t) \sim \mathfrak{N}(0, \sigma^2) iid \\ T_t = a + bt \quad \text{or} \quad T_t = a + e^{\alpha t} \end{cases} \quad (4)$$

where  $t = (1, 2, \dots, T_{obs})$ ,  $T_{obs} = 69120$ ,  $\beta_0 = 8$ ,  $\beta_1 = 0.8$ ,  $\beta_2 = 0.18$ ,  $\theta = 0.8$ ,  $\sigma^2 = 0.05$ ,  $a=100$ ,  $b \in \{0.01, 0.02, \dots, 0.05\}$  and  $\alpha \in \{0.001, 0.0011, \dots, 0.005\}$ .



a) one week simulated daily load



b) one year simulated daily peak load

**Fig. 1** Simulated daily power pattern

### 3.2 EMD trend extraction performance

To investigate the EMD trend extraction performance, a comparison with the non-parametric trend extraction method based on Hodrick-Prescott (HP) filter is performed. This last one is widely used by economists for trend estimation, see e.g. [6].

For a time series  $y = (y_1, y_2, \dots, y_{T_{obs}})$  supposed to contain a trend ( $T_t$ ) and a cyclical component ( $C_t$ ), the best extracted trend ( $\hat{T}_t$ ) is the solution of:

$$\min_{\{\hat{T}_t\}_{t=1}^{T-1}} \left\{ \sum_{t=1}^{T-1} (y_t - \hat{T}_t)^2 + \lambda \sum_{t=2}^{T-1} [(\hat{T}_{t+1} - \hat{T}_t) - (\hat{T}_t - \hat{T}_{t-1})]^2 \right\} \quad (5)$$

where the parameter  $\lambda$  is a positive number which penalizes variability in the growth rate of the trend component. The larger value of  $\lambda$ , the smoother the trend extracted and then a good extraction of a trend requires a suitably chosen value of  $\lambda$ , see [8] for theoretical investigation. Here we choose  $\lambda$  according to short empirical tuning based on simulated load curve for  $\lambda$  in the range  $10^2$  to  $10^{15}$ . In Table 1, are reported

| $\alpha$          |     | $Max\_MAE$ | $Max\_AE$ | Satisfactory HP $\lambda$ parameter range        |
|-------------------|-----|------------|-----------|--------------------------------------------------|
| $10^{-4}$         | HP  | 0.0125     | 0.0365    | $\lambda \in [10^9, 10^{11}]$                    |
|                   | EMD | 0.009      | 0.0317    |                                                  |
| $5 \cdot 10^{-4}$ | HP  | 0.0146     | 0.0463    | $\lambda \in [10^8, 3 \cdot 10^{11}]$            |
|                   | EMD | 0.02       | 0.1       |                                                  |
| $10^{-3}$         | HP  | 0.067      | 0.21      | $\lambda \in [1.09 \cdot 10^5, 3.25 \cdot 10^8]$ |
|                   | EMD | 0.022      | 0.06      |                                                  |
| $2 \cdot 10^{-3}$ | HP  | 0.052      | 0.41      | $\lambda \in [2.96 \cdot 10^5, 2.15 \cdot 10^8]$ |
|                   | EMD | 0.009      | 0.0317    |                                                  |

**Table 1** Performances of the HP and EMD simulated daily peak trend extraction

the Maximum of Mean Absolute Error ( $Max\_MAE^3$ ) and the Maximum of Absolute Error ( $Max\_AE$ ) estimated for the HP and the EMD trends extracted for different values of  $\alpha$ :  $10^{-4}$ ,  $5 \cdot 10^{-4}$ ,  $10^{-3}$  and  $2 \cdot 10^{-3}$ . Note that, these values are chosen in order to allow simulated trends covering linear, quasi linear and exponential trend shapes.

These first results show that the EMD-trend is very close to the optimal Hodrick-Prescott one and make the EMD as an effective alternative to trend extraction problem. Same results are also obtained through the EMD and a moving average filtering comparison. Indeed the EMD trends extracted are very close to those extracted through a conveniently chosen moving average filtering method. This expected finding is due to the adaptive nature of the principle of the EMD.

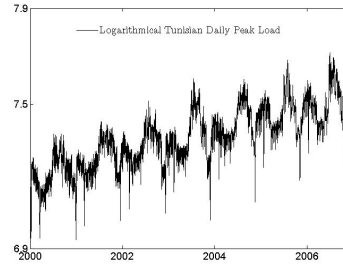
We notice in experiments not reported here, that high errors occur for high values of  $\alpha$  and that the end effects are so important for high value trends. In this case, the intrinsic trend can be obtained by aggregating the EMD residue and the last IMFs. It is also important to notice that there are various approaches to deal with the EMD end effect, for example by applying a window to the signal 'see [7]). Another solution is to extrapolate end maxima and end minima to construct the lower minima  $Imin_{i-1}$  and the upper maxima  $Imax_{i-1}$  envelopes (see [10]).

<sup>3</sup> For the EMD this statistic is reduced to the Mean Absolute Error  $MAE$

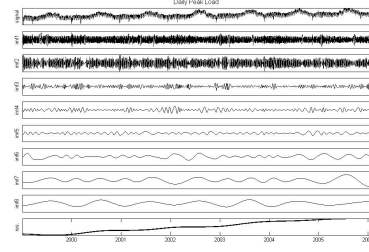
## 4 Real peak load time series

### 4.1 Peak load IMFs interpretation

We apply the EMD method to logarithmical Tunisian daily peak load from 2000 to 2006 (see Figure 2).



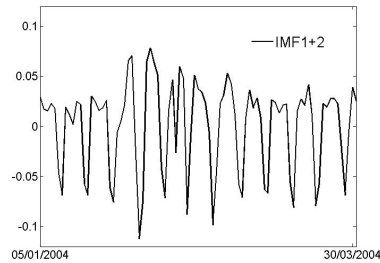
a) The logarithmical daily peak load 2000-2006



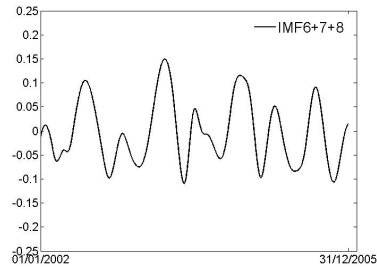
b) IMF components and the final residue or trend

**Fig. 2** EMD of the logarithmical daily peak load 2000-2006 from STEG utility

As previously noticed by Ould Mohamed Mahmoud et al. (2009), we note that IMF 1 to 2 exhibit high frequency and can represent very short term fluctuations (see Figure 3.a), IMF 3 to 5 capture small percentage of variance, indicating that such IMFs are not significant and finally IMF 6 to 8 capture mid-term effects described by seasonal variations (see Figure 3.b).



a) Sum of IMFs 1-2: weekly component

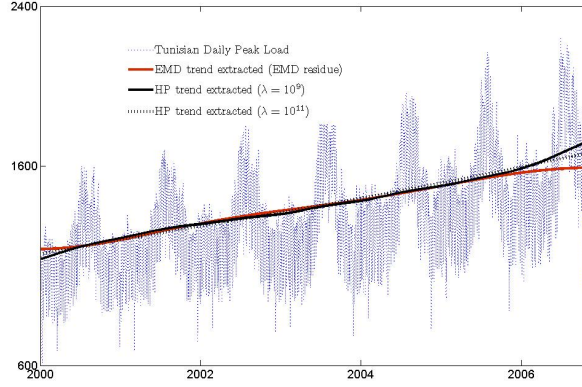


b) Sum of IMFs 6-7-8: annual component

**Fig. 3** IMFs connection to physical seasonal load components

## 4.2 Peak load trend extraction

The trend estimate is given by the residue component of the EMD. It could represent the major trend of long term load demand which may be related to economic growth in Tunisia. The results obtained for the two methods are given in Figure 5. One can find the EMD trend and two HP trends obtained from the two bounds of the tuning parameter ( $\lambda$ ) interval evidenced for linear or quasi-linear in section 3.2.



**Fig. 4** MA filtering, HP and EMD trends extracted of Tunisian daily peak load.

As previously mentioned, the results of trend extraction through the Hodrick-Prescott filter is very sensitive to the value of parameter  $\lambda$ . And this fine tuning make trend extraction more difficult. On the contrary, the EMD does not require any parameter choice depending on the analyzed data.

## 4.3 A remark about wavelets

Of course, wavelets are a natural competitive method for EMD. Let us recall quickly the hierarchical multiresolution structure of wavelet decomposition (see e.g. [4] for details). Wavelets allow to write a signal as the sum of orthogonal signals: a coarser approximation at large scale (low frequency) and additional details at different resolutions, of decreasing scales. The approximation at level  $j$  roughly represents the local mean signal on intervals of length  $2^j$  while the detail at level  $j$  contains fluctuations around this local mean on the same corresponding intervals. Let us consider a time series  $X$  of length  $n$ , then, for a given orthogonal wavelet  $\psi$ , the time series can be decomposed at level  $J$  (which is at most the integer part of  $\log_2(n)$ ) as:

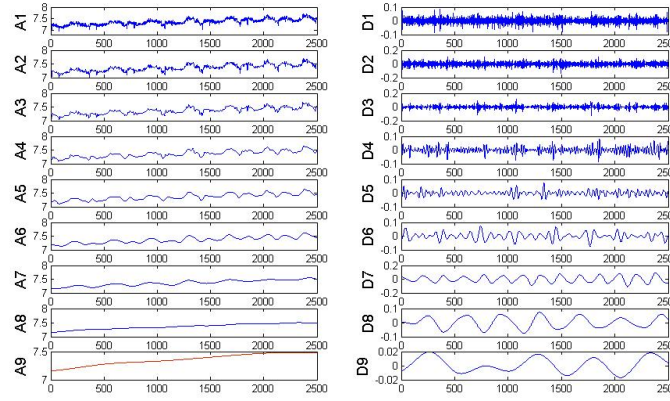
$$X = A_J + \sum_{j=1}^J D_j, \quad (6)$$

where  $A_j$  and  $D_j$  are respectively the approximation and detail at level  $j$  of the signal  $X$ .

For the wavelet usually denoted by daub5 (a regular compactly supported wavelet), the decomposition at level 9 of the previously considered electrical signal leads to Figure 5. On the left, the approximations from the finer (A1) to the coarser (A9). All these approximations are candidates to be the trends and the choice depends on the level of decomposition which acts as a tuning parameter related to the convenient scales to select. On the right, the details from the finer (D1) to the coarser (D9). Since

$$D_j = A_{j-1} - A_j, \quad (7)$$

each detail capture the difference between two successive approximations. It turns that A7, A8 or A9 are approximations which can be considered as good candidates to be trends.

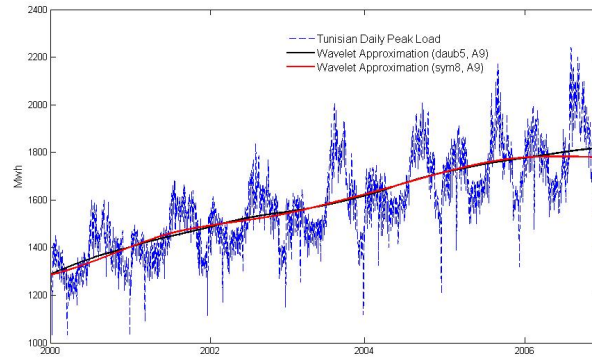


**Fig. 5** Tunisian daily peak load: daub5-wavelet approximations as candidates to be trends. A7, A8 or A9?

Since the level 9 seems to be convenient let us compare in Figure 6 the wavelet trends obtained for two different regular wavelets (daub5 and sym8).

As it can be seen the results are quite satisfactory when the decomposition level as well the wavelet are chosen carefully.

We emphasize that EMD trend extraction performs this directly by constructing a trend as a residue signal coming from an iterative process extracting oscillatory components. The trend is essentially the non-oscillatory component of the signal and the method does not require any tuning parameter or any wavelet choice thanks to its adaptive nature.



**Fig. 6** Tunisian daily peak load: level 9 wavelet trends for daub5 and sym8 wavelets

## 5 Conclusion

Empirical Mode Decomposition appears to be an eligible method for trend extraction from seasonal time series. This finding has been illustrated through the comparison of EMD trend extracted with an improved and widely used method in economics, based on HP Filter. Since EMD-trend is very close to the optimal Hodrick Prescott trend obtained after approximation of the optimal parameter of the filter, it turns out that EMD trend extraction method does not require any optimal tuning parameter thanks to its adaptive nature. In addition the final remark about a short wavelet example illustrates that wavelets, a natural competitive method for EMD, performs well but requires the wavelet choice and decomposition level.

## References

1. Alexandrov, T., Bianconcini, S., Bee Dagum, E., Maass, P. and Mc Elroy, T.: A Review of Some Modern Approaches to the Problem of Trend Extraction. In Research Report Series, Statistics 2008-3, U.S. Census Bureau, Washington (2009)
2. Flandrin, P., Goncalves, P. and Rilling, G.: Detrending and Denoising with Empirical Mode Decomposition. In EUSIPCO 2004. September 6-10, Vienna, Austria (2004)
3. Huang, N.E., Shen, Z., Long, S.R., Wu, M.C., Shih, H.H., Zheng, Q., Yen, N., Tung, C.C., and Liu, H.H.: The empirical mode decomposition and the Hilbert spectrum for nonlinear and nonstationary time series analysis. *Proceedings of the Royal Society London A*. **454**, 903995 (1998)
4. Misiti, M., Misiti, Y., Oppenheim, G., Poggi, J-M. Wavelets and their applications. Hermes Lavoisier, ISTE Publishing Knowledge (2007)
5. Ould Mohamed Mahmoud, M., Mhamdi, F. and Jaidane-Saidane, M. Long Term Multi-Scale Analysis of the Daily Peak Load Based on the Empirical Mode Decomposition. In: IEEE PowerTech, june 28-july 2, Romania (2009)

6. Pollock, D.S.G. . Sharp filters for short sequences. *Journal of Statistical Planning and Inference* **113**, 663–683 (2003)
7. Ren, D., Yang, S., Wu, Z. and Yan, G. . Evaluation of the EMD end effect and a window based method to improve EMD. In: *International Technology and Innovation Conference*, November 6-7, China (2006)
8. Schlicht, E. Estimating the smoothing parameter in the so-called Hodrick-Prescott filter. *J. Japan Statist. Soc.* **35**, 99–119 (2005)
9. Suling, J., Yanqin, G., Qiang, W. and Jian, Z. : Trend Extraction and Similarity Matching of Financial Time Series Based on EMD Method. In: *World Congress on Engineering and Computer Science*, 20-22 Oct 2009, San Francisco (2009)
10. Wu, Z., Huang, N.E., Long S.R. and Peng, C.K : On the trend, detrending, and variability of nonlinear and nonstationary time series. *PNAS* September 18, **104**, 38, 14889–14894 (2007)
11. Zhou, N., Trudnowski, D., Pierre, J.W., Sarawgi, S. and Bhatt, N. An algorithm for removing trends from power-system oscillation data. In: *IEEE PES*. July 20-24, Pittsburgh, PA (2008)