

Point and interval estimation for some financial performance measures

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Abstract In this paper we study the estimators of three financial performance measures: the Sharpe Ratio, the Gini Ratio and the Mean Absolute Deviation Ratio. The analysis is performed under two sets of assumptions. Firstly, the case of independent and identically normally distributed returns is considered. After that, relaxing the normality assumption, the case of independent and identically distributed returns is investigated. In both situations, we study the bias of the estimators and, in order to define confidence intervals, we derive their (exact or asymptotic) distributions.

Key words: financial performance measure, Sharpe Ratio, Gini Ratio, Mean Absolute Deviation Ratio, concentration measures, Statistical analysis of financial data

1 Introduction

Most of the known financial performance measures are defined as the ratio between a reward measure and a risk measure. For example, the well-known Sharpe Ratio, the Gini Ratio and the Mean Absolute Deviation Ratio (MAD Ratio) have this structure. For all these three indicators, the reward measure is the expected excess return (beyond some risk-free rate); while the risk measures differ among the indices. In particular, the risk is measured by the standard deviation of returns in the Sharpe Ratio, by the Gini mean difference of returns in the Gini Ratio and by the mean absolute deviation of returns in the MAD Ratio.

The Gini Ratio and the MAD Ratio were proposed to overcome the criticism against the use of the standard deviation as risk measure. The reason for this criticism is that the standard deviation is the natural risk measure only when returns

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are normally distributed. Since empirical evidence clearly shows that returns are not Gaussian, it can be considered appropriate to use other variability indicators for measuring the risk.

In this paper we derive a confidence interval for the three mentioned indices and we study the bias of their estimators. In detail, the paper is organized as follows. In section 2 the definitions of Sharpe Ratio, Gini Ratio and MAD Ratio are provided. In section 3 we analyze the case of independent and identically normally distributed returns (i.i.d.-normal returns) and in section 4 we consider the case of independent and identically distributed returns (i.i.d. returns). Section 5 is devoted to the conclusions.

In the following, we denote with F_H , μ_H and σ_H^2 the distribution function, the expectation and the variance of the r.v. H , respectively. The Gini's mean difference of H is denoted by Δ_H and the mean absolute deviation of H is denoted by δ_H . Finally, σ_{HK} stands for the covariance between the random variables H and K .

2 The Sharpe Ratio, the Gini Ratio and the MAD Ratio

Let X be the r.v. describing the return of a risky financial activity and let Y be the r.v. representing the return of the risk free financial activity. The r.v. $D = X - Y$ describes the excess return of the risky financial activity with respect to the risk free financial activity. The Sharpe Ratio (see [16] and [17]) is given by:

$$\psi^* = \frac{\mu_D}{\sqrt{\sigma_D^2}} = \frac{\mu_X - \mu_Y}{\sqrt{\sigma_X^2 + \sigma_Y^2 + 2\sigma_{XY}}} . \quad (1)$$

As pointed out in [17], this measure can be interpreted as the expected excess return per unit of risk where the risk is measured by the standard deviation of D .

Similarly, the Gini Ratio (see [15]) and the MAD Ratio (see [8]) are given by $\psi_\Delta^* = \mu_D/\Delta_D$ and $\psi_\delta^* = \mu_D/\delta_D$, respectively. The interpretation of these two performance measures are similar to that of ψ^* .

Even if the formula (1) is the definition of ψ proposed in [17], in literature the Sharpe Ratio is usually defined as $\psi = (\mu_X - \mu_Y)/\sigma_X$. That because, theoretically, the risk free financial activity has a constant return. Therefore, Y is a degenerate r.v. on μ_Y and $\psi^* \equiv \psi$. According to the most part of the literature, here we adopt this last “simplified” definition of Sharpe Ratio and we assume that the *risk free rate* μ_Y is known.

A similar simplification can be made for the Gini Ratio and the MAD Ratio. In particular, it is easy to verify that, for all $a \in \mathbb{R}$, $\Delta_{H+a} = \Delta_H$ and $\delta_{H+a} = \delta_H$. If Y is degenerate on μ_Y , it follows that $\Delta_D = \Delta_X$ and $\delta_D = \delta_X$. Consequently, the simplified definitions of the Gini Ratio and of the MAD Ratio are $\psi_\Delta = (\mu_X - \mu_Y)/\Delta_X$ and $\psi_\delta = (\mu_X - \mu_Y)/\delta_X$, respectively.

It can be noted that, under the assumption that μ_Y is a known constant, the just introduced simplified ratios depend only on the features of X . Then, to simplify the

notation, in the following we denote the risk free rate by ξ and we drop the subscript X in σ_X^2 , Δ_X , δ_X , and F_X .

3 The case of i.i.d.-Normal returns

Let X be a normal r.v. with parameters μ and σ : $X \sim \mathcal{N}(\mu, \sigma^2)$. In this case the Gini and the MAD Ratio can be expressed as a scale transformation of the Sharpe Ratio. In particular, it is well-known (see [14]) that $\Delta = 2\sigma/\sqrt{\pi}$ and $\delta = \sigma\sqrt{2/\pi}$. It then follows that

$$\psi_\Delta = \frac{\sqrt{\pi}}{2}\psi \quad \text{and} \quad \psi_\delta = \sqrt{\frac{\pi}{2}}\psi. \quad (2)$$

Let $X_1, X_2, \dots, X_n$ be an i.i.d. sample from F . In order to estimate the Sharpe Ratio it is natural to use the analogical estimator

$$\hat{\Psi} = \frac{(\bar{X} - \xi)}{\sqrt{S^2}}$$

where $\bar{X}$ and S^2 denote the sample mean and the unbiased sample variance. By the relations (2), the r.v. $\hat{\Psi}_\Delta^* = (\sqrt{\pi}/2)\hat{\Psi}$ is an estimator of the Gini Ratio. Similarly, the r.v. $\hat{\Psi}_\delta^* = \sqrt{\pi/2}\hat{\Psi}$ is an estimator of the MAD Ratio.

The properties of $\hat{\Psi}_\Delta^*$ and $\hat{\Psi}_\delta^*$ can be easily deduced from the features of $\hat{\Psi}$. For this reason, in the remainder of this section, we will mainly focus on $\hat{\Psi}$.

As first result it is possible to show that

$$\sqrt{n}\hat{\Psi} = \frac{\sqrt{n}\frac{\bar{X}-\mu}{\sigma} + \sqrt{n}\psi}{\sqrt{\left(\frac{(n-1)S^2}{\sigma^2}\right)/(n-1)}}. \quad (3)$$

From expression (3), it can be easily observed that the distribution of the r.v. $\sqrt{n}\hat{\Psi}$ is noncentral t with $(n-1)$ degrees of freedom and noncentrality parameter $\sqrt{n}\psi$. Thanks to that result, we can define an exact confidence interval for ψ . In detail, let $t_{v,p}(a)$ be the p -quantile of a noncentral t distribution with v degrees of freedom and noncentrality parameter a . Let $\hat{\psi}$ be an estimate of ψ . The extremes of the $(1-\alpha)$ -confidence interval $(\psi_-; \psi_+)$ for ψ are given by (see [1] and [7]):

$$\psi_- : t_{(n-1), \frac{\alpha}{2}}(\sqrt{n}\psi_-) = \sqrt{n} \cdot \hat{\psi}$$

$$\psi_+ : t_{(n-1), 1-\frac{\alpha}{2}}(\sqrt{n}\psi_+) = \sqrt{n} \cdot \hat{\psi}$$

The last two equations cannot be analytically solved but their solutions can be easily numerically computed. The confidence interval just introduced improves the one based on the asymptotic distribution of $\hat{\Psi}$ obtained in [6]. In [10] it is shown that

the estimator $\hat{\Psi}$ is biased and $E(\hat{\Psi}) = \psi \cdot d$ where

$$d = \sqrt{\frac{n-1}{2} \frac{\Gamma(\frac{n-2}{2})}{\Gamma(\frac{n-1}{2})}}.$$

In [2] it is proved that the bias factor d is greater than 1 for all $n > 2$. Then, the estimator $\hat{\Psi}_u = d^{-1}\hat{\Psi}$ is unbiased and it is more efficient than $\hat{\Psi}$.

An exact confidence interval for ψ_Δ and ψ_δ can be obtained by multiplying the extremes of the confidence interval for ψ by $\frac{\sqrt{\pi}}{2}$ and by $\sqrt{\frac{\pi}{2}}$, respectively. In addition, the estimators $\hat{\Psi}_{\Delta u}^* = d^{-1}\hat{\Psi}_\Delta^*$ and $\hat{\Psi}_{\delta u}^* = d^{-1}\hat{\Psi}_\delta^*$ are unbiased and more efficient than $\hat{\Psi}_\Delta^*$ and $\hat{\Psi}_\delta^*$, respectively.

4 The case of i.i.d. returns

The estimators for the Gini Ratio and the MAD Ratio proposed in the previous section stem from the particular relations existing among the standard deviation, the Gini mean difference and the mean absolute deviation of a normal r.v.. In this section, we do not specify a particular parametric model for F . Consequently, the estimators of ψ_Δ and ψ_δ cannot be defined starting from the estimators of the parameters of F . Then, we consider the analogical estimators

$$\hat{\Psi}_\Delta = \frac{\bar{X} - \xi}{\hat{\Delta}} \quad \text{and} \quad \hat{\Psi}_\delta = \frac{\bar{X} - \xi}{\hat{\delta}}$$

where

$$\hat{\Delta} = \frac{1}{n(n-1)} \sum_{i=1}^n \sum_{j=1}^n |X_i - X_j| \quad \text{and} \quad \hat{\delta} = \frac{1}{n} \sum_{i=1}^n |X_i - \bar{X}|.$$

The exact distributions of $\hat{\Psi}$, $\hat{\Psi}_\Delta$, and $\hat{\Psi}_\delta$ cannot be derived because F is unknown. However their limit distribution can be obtained. For that purpose, it is useful to note that the Gini Ratio and the MAD Ratio are strictly related to two well-known concentration measures. In more detail, $\psi_\Delta = (2G)^{-1}$ and $\psi_\delta = (2P)^{-1}$ where $G = \Delta/(2\mu)$ and $P = \delta/(2\mu)$ are the Gini concentration ratio and the Pietra concentration ratio of the r.v. $X - \xi$. Consequently, several results concerning the estimators of G and P can be used in the study of $\hat{\Psi}_\Delta$ and $\hat{\Psi}_\delta$.

Limit distribution of $\hat{\Psi}$

Let $X_1, \dots, X_n$ be an i.i.d. sample from F and assume that $E[X^4] < \infty$. It is known that

$$\sqrt{n} \begin{bmatrix} \bar{X} - \mu \\ S^2 - \sigma^2 \end{bmatrix} \xrightarrow{d} \mathbf{NB} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \sigma^2 & \mu_3 \\ \mu_3 & \mu_4 - \sigma^4 \end{bmatrix} \right) \quad (4)$$

where $\mu_k = E[(X - \mu)^k]$ and **NB** means “bivariate normal”. Starting from result (4), Lo ([9]) derived the limit distribution of $\hat{\Psi}$ by the *delta-method*:

$$\sqrt{n}(\hat{\Psi} - \psi) \xrightarrow{d} \mathcal{N}(0; V) \quad \text{where} \quad V = 1 - \frac{\mu_3}{\sigma^3} \psi + \left(\frac{\mu_4}{\sigma^4} - 1 \right) \frac{\psi^2}{4}. \quad (5)$$

The variance V can be consistently estimated by $\hat{V} = 1 - \frac{\hat{\mu}_3}{\hat{\sigma}^3} \hat{\Psi} + \left(\frac{\hat{\mu}_4}{\hat{\sigma}^4} - 1 \right) \frac{\hat{\Psi}^2}{4}$ where $\hat{\mu}_3$ and $\hat{\mu}_4$ denote the third and the fourth sample central moment. The asymptotic $(1 - \alpha)$ -confidence interval for ψ is

$$\left(\hat{\Psi} - z_{1-\frac{\alpha}{2}} \sqrt{\hat{V}/n}; \hat{\Psi} + z_{1-\frac{\alpha}{2}} \sqrt{\hat{V}/n} \right). \quad (6)$$

Limit distribution of $\hat{\Psi}_\Delta$

In order to obtain the limit distribution of $\hat{\Psi}_\Delta$, we recall the following result, due to Hoeffding ([5]).

Let $X_1, \dots, X_n$ be an i.i.d. sample from F and assume that $E[X^2] < \infty$. Then

$$\sqrt{n} \begin{bmatrix} \bar{X} - \mu \\ \hat{\Delta} - \Delta \end{bmatrix} \xrightarrow{d} \mathbf{NB} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \sigma^2 & \gamma \\ \gamma & \zeta^2 \end{bmatrix} \right) \quad (7)$$

where $\gamma = 2(\mathcal{D} - \mu\Delta)$, $\zeta^2 = 4(\mathcal{F} - \Delta^2)$ and

$$\mathcal{D} = \int_{-\infty}^{\infty} x|x-y|dF(y)dF(x), \quad \mathcal{F} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x-y||x-z|dF(z)dF(y)dF(x).$$

In [5] this result is used to derive the limit distribution of the Gini Concentration Ratio. Here, starting from (7), the limit distribution of $\hat{\Psi}_\Delta$ is obtained by the *delta-method*:

$$\sqrt{n}(\hat{\Psi}_\Delta - \psi_\Delta) \xrightarrow{d} \mathcal{N}(0; V_\Delta) \quad \text{where} \quad V_\Delta = \frac{\sigma^2}{\Delta^2} - 2\frac{\gamma}{\Delta^2} \psi_\Delta + \frac{\zeta^2}{\Delta^2} \psi_\Delta^2. \quad (8)$$

An unbiased and consistent estimator for ζ^2 is given by (see [18]):

$$\hat{\zeta}^2 = \frac{4}{(n-1)} \left[S^2 + (n-2)\hat{\mathcal{F}} - \frac{(2n-3)}{2}\hat{\delta}^2 \right]$$

where

$$\hat{\mathcal{F}} = \frac{1}{n(n-1)(n-2)} \sum_{i=1}^n \sum_{j=1}^n \sum_{l=1}^n |X_i - X_j||X_i - X_l| - \frac{2S^2}{n-2}.$$

An unbiased and consistent estimator for γ is given by (see [13]):

$$\hat{\gamma} = 2[\hat{\mathcal{D}} - \bar{X}\hat{\Delta}] \quad \text{where} \quad \hat{\mathcal{D}} = \frac{1}{n(n-1)} \sum_{i=1}^n \sum_{j=1}^n X_i |X_i - X_j|.$$

So, the variance V_Δ can be consistently estimated by $\widehat{V}_\Delta = \frac{s^2}{\Delta^2} - 2\frac{\hat{\gamma}}{\Delta^2}\psi_\Delta + \frac{\hat{\xi}^2}{\Delta^2}\widehat{\Psi}_\Delta^2$ and the following asymptotic $(1 - \alpha)$ -confidence interval for ψ_Δ can be obtained:

$$\left(\widehat{\Psi}_\Delta - z_{1-\frac{\alpha}{2}} \sqrt{\widehat{V}_\Delta/n}; \widehat{\Psi}_\Delta + z_{1-\frac{\alpha}{2}} \sqrt{\widehat{V}_\Delta/n} \right).$$

Limit distribution of $\widehat{\Psi}_\delta$

The limit distribution of $\widehat{\Psi}_\delta$ can be obtained by the *delta-method* starting from the following result, due to Gastwirth ([3]).

Let $X_1, \dots, X_n$ be an i.i.d. sample from F and assume that $E[X^2] < \infty$. Then

$$\sqrt{n} \begin{bmatrix} \bar{X} - \mu \\ \hat{\delta}^2 - \delta \end{bmatrix} \xrightarrow{d} \mathbf{NB} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \sigma^2 & \kappa \\ \kappa & v^2 \end{bmatrix} \right)$$

where

$$v^2 = 4p^2\sigma^2 + 4(1-2p) \int_{-\infty}^{\mu} (x-\mu)^2 dF(x) - \delta^2,$$

$$\kappa = 2p\sigma^2 - 2 \int_{-\infty}^{\mu} (x-\mu)^2 dF(x), \quad \text{and} \quad p = F(\mu).$$

Gastwirth ([3]) use this result to obtain the limit distribution of the Pietra concentration ratio. In the same way we obtain the limit distribution of $\widehat{\Psi}_\delta$:

$$\sqrt{n}(\widehat{\Psi}_\delta - \delta) \xrightarrow{d} \mathcal{N}(0; V_\delta) \quad \text{where} \quad V_\delta = \frac{\sigma^2}{\delta^2} - 2\frac{\kappa}{\delta^2}\psi_\delta + \frac{v^2}{\delta^2}\psi_\delta^2.$$

A consistent estimator of v^2 is given by $\hat{v}^2 = (4\hat{p}\hat{S}^2 + 4(1-2\hat{p})\hat{\mu}_2^-)$ where

$$\hat{p} = \frac{1}{n} \sum_{i=1}^n I_i, \quad \hat{\mu}_2^- = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2 I_i \quad \text{and} \quad I_i = \begin{cases} 1, & X_i < \bar{X} \\ 0, & \text{otherwise} \end{cases}.$$

A consistent estimator of κ is given by $\hat{\kappa} = (2\hat{p}\hat{S}^2 - 2\hat{\mu}_2^-)$.

Consequently, $\widehat{V}_\delta = \frac{\hat{S}^2}{\hat{\delta}^2} - 2\frac{\hat{\kappa}}{\hat{\delta}^2}\widehat{\Psi}_\delta + \frac{\hat{v}^2}{\hat{\delta}^2}\widehat{\Psi}_\delta^2$ is consistent for V_δ and

$$\left(\widehat{\Psi}_\delta - z_{1-\frac{\alpha}{2}} \sqrt{\widehat{V}_\delta/n}; \widehat{\Psi}_\delta + z_{1-\frac{\alpha}{2}} \sqrt{\widehat{V}_\delta/n} \right)$$

is the asymptotic $(1 - \alpha)$ -confidence interval for ψ_δ .

Note that it is necessary to assume the existence of the fourth moment of X in order to derive the limit distribution of $\widehat{\Psi}$ while it is only necessary to assume the existence of the second moment of X to obtain the limit distributions of $\widehat{\Psi}_\Delta$ and $\widehat{\Psi}_\delta$. This observation is important because empirical evidence suggests that the distribution of the returns of many financial activities have infinite fourth moment

(especially when high frequency returns are considered, see [4]). In these cases the confidence intervals (6) cannot be used. This is a potential limitation of ψ .

The estimators $\widehat{\Psi}$, $\widehat{\Psi}_\Delta$, and $\widehat{\Psi}_\delta$ are biased. In order to quantify the bias, we recall the following Taylor series approximation of the function $f(x, y) = x/y$ around the point (a, b) :

$$\frac{x}{y} \approx \frac{a}{b} + \frac{1}{b}(x-a) + \frac{a}{b^2}(y-b) - \frac{1}{b^2}(x-a)(y-b) + \frac{a}{b^3}(y-b)^2. \quad (9)$$

Let $(a, b) = (\mu - \xi, \sigma)$. Using the expression (9) and observing that

$$E[S] \approx \sigma - \frac{1}{8n} \left(\frac{\mu_4}{\sigma^4} - \frac{n-3}{n-1} \right) \quad \text{and} \quad \sigma_{\bar{X}S} \approx \frac{\mu_3}{2n\sigma},$$

the following approximation of $E[\widehat{\Psi}]$ holds:

$$\begin{aligned} E \left[\frac{\bar{X} - \xi}{S} \right] &\approx E \left[\psi + \frac{\bar{X} - \mu}{\sigma} + \psi \left(\frac{S - \sigma}{\sigma} \right) + \frac{(\bar{X} - \mu)(S - \sigma)}{\sigma^2} + \psi \left(\frac{S - \sigma}{\sigma} \right)^2 \right] \\ &= \psi \left[1 + \frac{1}{4(n-1)} + \frac{1}{8n} \left(\frac{\mu_4}{\sigma^4} - 3 \right) \right] - \frac{1}{2n} \frac{\mu_3}{\sigma^3}. \end{aligned} \quad (10)$$

In a similar way, the following approximations can be obtained:

$$E[\widehat{\Psi}_\Delta] \approx \psi_\Delta \left(1 + \frac{\xi^2}{n\Delta^2} \right) - \frac{\gamma}{n\Delta^2}; \quad E[\widehat{\Psi}_\delta] \approx \psi_\delta \left(1 + \frac{v^2}{n\delta^2} \right) - \frac{\kappa}{n\delta^2}. \quad (11)$$

The approximations (10) and (11) can be used to define approximatively unbiased estimators for ψ , ψ_Δ , and ψ_δ . For example, the estimator

$$\widehat{\Psi}_{\Delta u} = \left(\widehat{\Psi}_\Delta + \frac{\hat{\gamma}}{n\hat{\Delta}^2} \right) \left(1 + \frac{\hat{\xi}^2}{n\hat{\Delta}^2} \right)^{-1}$$

is approximatively unbiased for ψ_Δ . However, the variability of $\widehat{\Psi}_{\Delta u}$ could be much higher than the variability of $\widehat{\Psi}_\Delta$, because of the variability of the estimators $\hat{\gamma}$, $\hat{\Delta}$, and $\hat{\xi}^2$. So, a further study to compare the mean square errors of $\widehat{\Psi}_{\Delta u}$ and $\widehat{\Psi}_\Delta$ is necessary. Similar observation can be made for the others two indices.

5 Conclusion

In this paper we study some inferential aspects of the Sharpe Ratio, the Gini Ratio and the MAD Ratio. Under the assumption of i.i.d.-normal returns, exact confidence intervals for the three indices are obtained. After that, recalling the result of Miller and Gher ([10]), the unbiased estimators of the indicators are provided. Further,

we show that the biased-corrected estimators turn out to be more efficient than the biased ones. Under the assumption of i.i.d. returns, also using some well-known results concerning the inferential aspects of the Gini Concentration Ratio and the Pietra concentration Ratio, we derive the asymptotic confidence intervals for the Sharpe, Gini and MAD Ratio. Finally, approximatively unbiased estimators for the three indices are introduced also in the context of i.i.d. returns. In a further study we will examine the efficiency of these bias-corrected estimators with respect to the biased ones and we will examine the coverage accuracy and the length of the asymptotic confidence intervals in finite samples.

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