

Object-Oriented Bayesian Networks for firms' collusion strategies

Julia Mortera, Cecilia Vergari and Paola Vicard

Abstract This paper shows how Object-Oriented Bayesian Networks can be used to model a duopolist decision process integrated with external market information. Both the relational structure and the parameters of the market behaviour model are estimated (learned) from a real dataset. Various decision scenarios are shown and discussed.

Key words: Bayesian networks, prisoner's dilemma, repeated games, structural learning

1 Introduction to the Italian Antitrust Authority decision process

Firms in many cases have incentives to cooperate (collude) to increase their profits. The Antitrust Authority's (AA) main task is to monitor and to prevent potential anti-competitive behaviour and their effects. Here the AA decision process is modelled via a Bayesian network (BN) [2] estimated from real data. We study how the AA monitoring affects firms' strategies about cooperation. Firms' strategies are modelled as a repeated prisoner's dilemma using object-oriented Bayesian networks (OOBNs) (see [5] and [1]).

In this application the actors are: the duopolists (firm 1 and firm 2) and the Antitrust Authority. Figure 1 shows the global model, where both the duopoly and the AA's decision process are represented by OOBNs. The Duopoly network is an influence diagram (ID) ([4]), *i.e.* a BN with decision and utility nodes, and represents the

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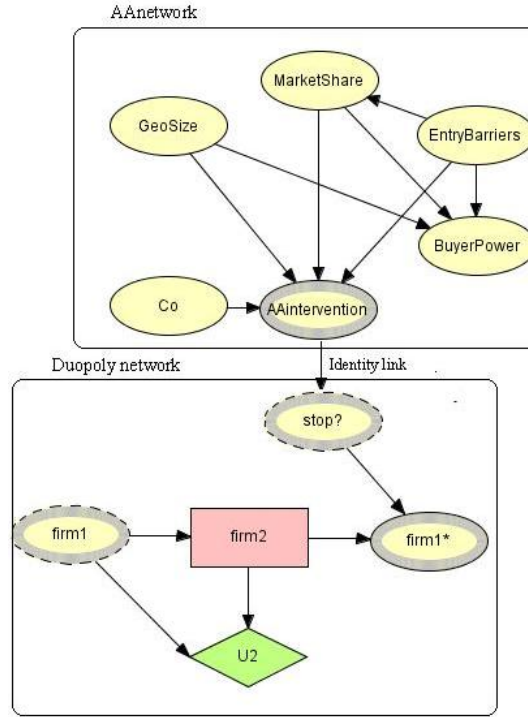


Fig. 1 Global model for a collusion stage game.

one-stage prisoner's dilemma between two firms producing identical goods. The AA network represents the decision process of the AA and is estimated from real data. In this paper we show how these two networks are derived and used as a decision support system.

In Figure 1 random variables are represented as oval nodes, decisions are represented as rectangular nodes and a rhombus is used for the utility.

2 Duopoly network

The duopoly network¹ models the one-stage game of a repeated prisoner's dilemma between two firms competing in the market. As consumers will buy from the firm charging the lowest price, firms have the incentive to undercut their price to conquer the market; as a result at equilibrium they will set the competitive price, gaining zero profits. However, if they decide to cooperate and set the monopoly price they could share positive monopoly profits. The problem is that this cooperative behaviour is

¹ We use the software HUGIN 6.9 (www.hugin.com) to implement our examples.

not credible when they interact only once. However, repeated interactions between duopolists should facilitate collusion.

The simultaneity of the game is represented by **firm1** being a random variable, whereas **firm2** is the decision maker, having two possible actions *cooperate* (1) and *defect* (0). Firm 2's optimal choice is influenced by firm 1 whose associated probability distribution represents firm 2's uncertainty about firm 1's behaviour. Random node **firm1** has two states defect (0) and cooperate (1) with uniform prior probabilities indicating firm 2's ignorance about firm 1's choice. Table 1 shows the utility (node U2) of **firm2** associated to **firm1** and **firm2**'s actions. U2 corresponds to the payoff matrix of a prisoner's dilemma. In each stage the game can either continue or terminate. The node **firm1*** models the behaviour of **firm1** in the

Table 1 Utility U2 for **firm1** and **firm2**'s actions.

firm1	cooperate	defect
firm2	cooperate	defect
U2	2	3
	0	1

next stage. Since in a repeated game every stage depends on the actions taken in the previous stages, **firm1*** is a child of **firm2**. Node **stop?** models the probability that the future stage game takes place. Table 2 gives the conditional probability distribution of **firm1*** given **stop?** and **firm2**. It shows that if the game stops, *i.e.* **stop?**=1, **firm1*** stops (2) with certainty, else **firm1*** cooperates (1) or defects (0) according to firm 2's decision. This implements the *tit for tat* (TFT) strategy, *i.e.* firm 1 begins by cooperating and cooperates as long as firm 2 cooperates and defects otherwise.

Table 2 Conditional probability table for **firm1*** given **stop?** and **firm2**.

stop?	0		1	
firm2	defect	cooperate	defect	cooperate
defect (0)	1	0	0	0
cooperate (1)	0	1	0	0
stop (2)	0	0	1	1

Since the game is symmetric, firm 2's optimal strategy coincides with firm 1's optimal strategy and this pair of strategies constitutes a Nash equilibrium, thus the choice of **firm2** as decision maker is without loss of generality.

3 Antitrust Authority network

The estimation (learning) process of a BN consists of two phases: the graphical structure estimation and the probability table estimation.

The data we use (described in [6]) were collected from 1994 to 2003 by the Italian Antitrust Authority. We examine a subset of this data relative to the “Petrochemical” economic sector. This dataset consists of 1010 observations on the variables described in Table 3.

Table 3 Variables for the AA network.

Variable	States	Description
Co	{ <i>Amato, Tesauro</i> }	head of the AA commission
MarketShare	{< 20%, 20 – 40%, > 40%}	post-collusion market share
EntryBarriers	{0, 1}	absence/presence of entry barriers (0/1)
BuyerPower	{0, 1}	weak/strong buyer power (0/1)
GeoSize	{ <i>sub – nat, nat, int</i> } ^a	size of the market of interest
AAintervention	{0, 1}	AA intervenes no/yes (0/1)

^a sub-national, national, international

3.1 Estimation of the graphical structure

The graphical representation of the AA decision process (AAnetwork in Figure 1) is obtained by a combination of subject-matter knowledge, provided by a domain expert, and the information coming from the data.

The *Necessary Path Condition* (NPC) algorithm ([7]) implemented in Hugin is used to estimate the graphical structure of the network. The NPC algorithm also takes into account constraints such as presence/absence of a link or its direction. Here we impose that if node **AAintervention** is connected with any of the other variables, the direction has to be from these into **AAintervention**. The dependencies estimated from data are now illustrated.

- **AAintervention** directly depends on the commission in charge of the case (**Co**), on the geographical size of the market (**GeoSize**), on the post-collusion market share (**MarketShare**) and on the existence of entry barriers (**EntryBarriers**). This implies that in a market without entry barriers, two firms having low market shares are likely to obtain the authorization to collude, because a market with these features is generally characterized by a high degree of competition.
- **BuyerPower** is directly affected by **GeoSize**, **MarketShare** and **EntryBarriers**. For instance, for larger geographical size the presence of buyer

power is more likely since presumably firms competing at international level have low bargaining power with respect to buyers.

- **MarketShare** directly depends on **EntryBarriers**. This is reasonable since whenever a market sector is characterized by entry barriers it is composed by a few firms with high market shares.

3.2 Estimation of the probability tables

In order to complete the construction of our model, we need to estimate the conditional probability distributions from the data. The EM-algorithm ([3]) is used for learning probabilities. Our sample is complete, so the algorithm reduces to a single iteration. However, this methodology efficiently deals with incomplete datasets as well.

The highlighted part of Figure 4A displays the marginal probabilities estimated from our data. The probability of an AA intervention is only 0.0199. This is most likely due to the fact that the probability that the market share is less than 20% is 0.6123 and that entry barriers are absent is 0.9753.

3.3 Using the network

Once the model has been estimated, we can address a number of questions about the AA's decision process. Various possible scenarios can be studied by inserting and propagating the appropriate evidence throughout the network. We illustrate two hypothetical scenarios in Figure 2.

Scenario A. What is the probability of authority intervention in a merger request when a firm has international market geographical size? By inserting and propagating the evidence **GeoSize** = international, we obtain a posterior probability for **AA intervention** equal to 0.0086 (Figure 2A).

The network can be used not only for direct reasoning about the probability of **AAintervention**, but also for indirect reasoning about the most plausible factors that influence a given AA decision.

Scenario B. Suppose that the AA decides to intervene in a firm's merger request. What are the most plausible reasons for this decision? Figure 2B gives the posterior probabilities given **AAintervention**=1. On comparing the highlighted part of Figure 4A and Figure 2B, we see, for example, that the probability of **EntryBarriers**= 1 increases dramatically from 0.0247 to 0.5431 and that the probability of **MarketShare**> 40% increases from 0.0969 to 0.3902.

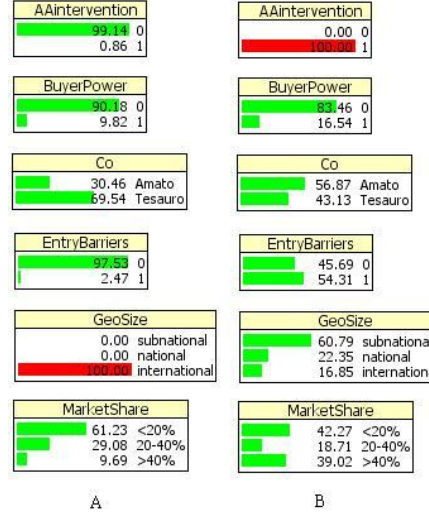


Fig. 2 Scenarios A and B giving marginal posterior percentages for AA network.

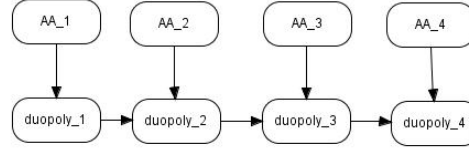


Fig. 3 OOBN representing a three-stage collusion game with uncertainty about the number of stages.

4 Global network

Figure 1 combines the AA and the Duopoly networks. Based on a real dataset the AA network models the probability that the game continues to a further stage (represented by AAintervention in AA network identical to stop? in Duopoly network). The two networks can be linked thanks to the modularity and flexibility of OOBN architecture.

Since firms interact more than once in a duopoly model and, moreover, the AA decision process is usually dynamic due to evolving market conditions and antitrust laws, we need to represent a repeated version of the model in Figure 1.

Figure 3 represents the global model (Figure 1) repeated four times for a three-stage collusion game with uncertainty on the number of stages. In this model, the AA's decision process is represented by the same network model in each period. Nevertheless, changes in the probability distributions which reflect an evolving market and changes in the laws, can be incorporated in the global network.

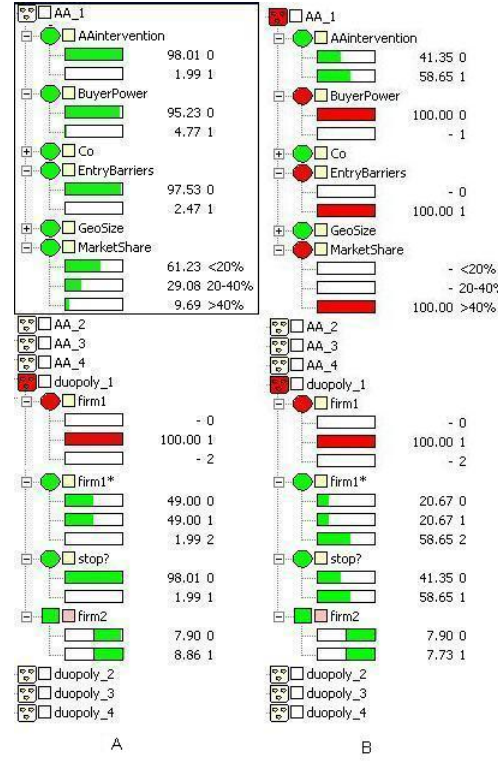


Fig. 4 Marginal probabilities and optimal decision of the first stage of the repeated collusion game when firm 1 plays TFT.

4.1 Firms' strategy

We now study the sensitivity of cooperative behaviours with respect to market sector, geographical size and market share. Figure 4 illustrates scenarios associated to the tit for tat (TFT) strategy and shows the marginal probabilities for a selection of random variables and the expected utilities for the decision nodes in the first stage subnetworks AA.1 and duopoly.1.

Figure 4A shows that firm 2's expected utility is 8.86 for cooperating and 7.90 for defecting. Therefore when no evidence about the market is included, firm 2's optimal decision in the first stage is to cooperate. This result can be partially explained by the high value, 0.9801, of the probability of non-intervention (which is equivalent to the probability that the game continues).

We examine how the optimal decision of firm 2 in the first stage changes when there are: entry barriers, no buyer power and the post-collusion market share is greater than 40%. In Figure 4B, we can see that inserting and propagating this evidence, *i.e.* EntryBarriers=1, BuyerPower=0 and MarketShare= > 40%,

in AA_1, the estimated probability of non-intervention decreases from 0.9801 to 0.4135. This in turn induces `firm 2` to optimally defect (the expected utility to defect is 7.90 while to cooperate is now 7.73).

These are illustrative examples; numerous different questions can be solved and various different strategies can be implemented using this same OOBN. Clearly the number of possible strategies envisaged increases with the number of game stages.

5 Discussion

Here we have presented an OOBN model for a repeated collusion game accounting for uncertainty in AA's behaviour. OOBNs appear to be a very promising tool in this context. On one hand they can represent and solve a repeated game with a computational complexity growing linearly with the number of repetitions (stages). Whereas, in traditional decision tree representations complexity grows exponentially. On the other hand, thanks to the modularity of OOBNs, it is possible to integrate the game representation with a complex statistical model. This gives rise to a global, coherent and easily interpretable tool to support decisions in complex problem domains.

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