

Modeling and Forecasting Realized Range Volatility

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Abstract In this paper, we estimate, model and forecast Realized Range Volatility, a new realized measure and estimator of the quadratic variation of financial prices. This estimator was early introduced in the literature and it is based on the high-low range observed at high frequency during the day. We consider the impact of the microstructure noise in high frequency data and correct our estimations, following a known procedure. Then, we model the Realized Range accounting for the well-known stylized effects present in financial data. We consider an HAR model with asymmetric effects with respect to the volatility and return and a GARCH and GJR-GARCH specification for the variance equation of our model. Moreover, we also consider a non Gaussian distribution for the residuals of our model. Analysis of the forecast performance of the different models in the different horizons suggest diverse results.

Key words: Statistical analysis of financial data, Econometrics, Forecasting methods, Time series analysis, Realized Range Volatility, Realized Volatility, Long-memory, Volatility forecasting

1 Introduction

In the last years, realized volatility measures, constructed from high frequency financial data and modeled with standard time series techniques, have shown to perform much better than traditional generalized autoregressive conditional heteroskedastic-

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ity (GARCH) and stochastic volatility models, when forecasting conditional second order moments. Most of the works that forecast volatility through realized measure, have concentrated on the *Realized Variance* (*RV*) introduced by Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002). The *RV* is based on the continuous time price theory and it is defined as a function of the sum of squared intraday returns. Basically, the *RV* is a highly efficient and unbiased estimator of the quadratic variation and converges to it when the intraday period goes to zero. Later on, Martens and van Dijk (2007) and Christensen and Podolskij (2007) introduced the *Realized Range Volatility* (*RRV*), another realized estimator consistent for the quadratic variation. The *RRV* is based on the difference between the minimum and maximum prices observed during a certain time interval. This new estimator tries to exploit the higher efficiency of the range relatively to that of the squared daily close-to-close return in the estimation of quadratic variation.

When dealing with high frequency financial market data, the asymptotic properties of the simple estimators are highly affected by the microstructure noise (non continuous trading, infrequent trade, bid ask bounce). As a result, an important part of the literature has presented different corrections to restore the efficiency of realized estimators for the volatility. These studies aimed at improving over the first generation of models, whose purpose was to construct estimates of realized variances by using series at a moderate frequency (see Andersen et al. (2003)). Some of the corrections presented to the *RV* are the Two Time Scale Estimator (TTSE), the sub-sampling method of Zhang et al. (2005), the generalization introduced by Zhang (2006). We also mention the approach for identifying the optimal sampling frequency by Bandi and Russell (2008), through a minimization of the MSE. Furthermore, kernel estimation was introduced by Hansen and Lunde (2006), while Barndorff-Nielsen et al. (2008) provide a generalization of this approach. Differently, Martens and van Dijk (2007) proposed a correction for the *RRV* based on scaling the range with the daily range and Christensen et al. (2009) presented another approach based on an adjustment by a constant which has to be estimated by simulation methods.

With the availability of new observable series for the volatility, many authors have applied traditional discrete time series models for their forecast (and implicitly for the forecast of returns volatility). Financial data are characterized by a series of well-known stylized facts. Being able to capture them, will result in a more accurate prevision of our variable of interest. These stylized facts are also observable over realized variance series and require appropriate modeling strategies. The presence of long-memory in volatility, documented in several studies, has been modeled through different specification: Andersen et al. (2003) introduced an ARFIMA model, and their forecasts for the *RV* generally dominate those obtained through GARCH models; Corsi (2009) presented the Heterogenous autoregressive (HAR) model, that reproduces the hyperbolic decay of the autocorrelation function by including the sums of *RV* over different horizons in order to capture the time strategies of the agents in the market. The second model has the advantage to be much simpler to estimate. Additionally, asymmetric, leverage effects, and fat tails should also be taken into account. Martens et al. (2009) specified a flexible unrestricted high-

order AR model. They also considered leverage effects, days of the week effects and macroeconomics announcement. Differently, Corsi et al. (2008) presented a HAR model and they introduced two important extensions specifying a GARCH component modeling the volatility of volatility and assuming non Gaussian errors. Their results suggested an improvement in the accuracy in the point forecasting and a better density forecast.

In this work, we model and forecast volatility through the *Realized Range Volatility*. Our main objective is to study the prediction performance of the range as a proxy of the volatility. An accurate forecast of financial variability should have important implication in asset and derivative pricing, asset allocation, and risk management. Moreover, we try to fill a gap in the literature comparing the performance of the realized range with the more common realized volatility. In the first part of this paper we construct and analyze the realized range series, correct it from the microstructure noise following Martens and van Dijk (2007). In the second part, we implement time series techniques to model and capture the stylized facts within the volatility equation to gain in forecasting accuracy. In details, we consider an HAR model, we introduce leverage effects with respect to the return and the volatility, and a GARCH a GJR-GARCH specification for the volatility of volatility. Furthermore, in order to capture the statistical feature of the residuals of our model, we also consider a Normal Inverse Gaussian (NIG) distribution. The remainder of this paper is structured as follows. In section 2, we present the data and the correction procedure. In section 3, we present the model and we discuss the results for the estimation and forecast in section 4. Finally, section 5 presents the results and futures steps.

2 Data and correction procedure

Under the assumption that there are no market frictions and there is continuous trading, the *RRV* is five time more efficient than *RV*. In the reality, there are evidences against these assumptions and realized estimator became inconsistent and unbiased. Hence, a corrected version for the *RRV* should restore the efficiency of this estimator over the *RV*. In this paper, we follow Martens and van Dijk (2007) that proposed a correction based on scaling the range with the daily range. Basically, the scaling bias correction is not difficult to implement it does not require the availability of tick by tick data. The idea of Martens and van Dijk (2007) is based on the fact that the *daily range* is almost not contaminated by market frictions. The simulation results of Martens and van Dijk (2007) confirm the theory that the range is more efficient than the *RV* and in the presence of market frictions the scaling correction removes the bias and restores the efficiency of the Realized Range estimator over the Realized Volatility. The *RRV* is defined as

$$RRV_t^\Delta = \frac{1}{\lambda^2} \sum_{i=1}^n (\ln p_{t,\Delta}^{hg} - \ln p_{t,\Delta}^{lo})^2$$

where $p_{t,\Delta}^{hg}$ and $p_{t,\Delta}^{lo}$ are the high and low prices for day t , λ is a scaling factor and Δ indicates the sampling frequency. Therefore, the *scaled RRV* is defined as:

$$RRV_{scaled,t}^{\Delta} = \left(\frac{\sum_{l=1}^q RRV_{t-l}^{\Delta}}{\sum_{l=1}^q RRV_{t-l}^{\Delta}} \right) RRV_t^{\Delta}$$

where q is the number of previous trading days used to compute the scaling factor. If the trading intensity and the spread do not change, q must be set as large as possible. However, in the reality only recent history should be taking into consideration.

Our database consists in five year of high, low, open and close prices sampled at 1 minute of Procter & Gamble Company (full paper will contain results over 12 companies). The original sample covers the period from January 2, 2003 to December 31, 2008, from 09:30 trough 16:00 and a total of 1,553 trading days. We constructed the series for the range for the one, five, thirty minutes and daily sampling frequency. We correct them on one, two and three previous months (i.e. 22, 44 or 66 days). The results of the corrections show that, after scaling, the volatility stabilized across the different sampling frequencies and scaled factors. Finally, we choose to sample every five minutes and to correct with the 66 previous days, the same election of the authors. A statistical analysis of the return and volatility series confirms the presence of the stylized facts vastly documented in the literature. The distribution of the returns exhibit excess of kurtosis while the square returns presents a slow decay in the autocorrelation function. The long-memory pattern in the hyperbolic and slowly decay of the ACF is much more pronounced for the *RRV* series.

3 Models for the observed volatility sequences

Different models have been presented to capture the stylized facts that financial series exhibit. Based on the statistical features briefly mentioned before, we consider the HAR model of Corsi (2009) to capture the long-memory pattern. We account for asymmetric effects with respect to the volatility and the returns. Moreover, following Corsi et al. (2008) we also include a GARCH specification to account for heteroskedasticity in observed volatility sequences and a standardized Normal Inverse Gaussian (NIG) distribution to deal with the observed skewness of the residuals. Finally, to account for asymmetric effects in the variance equation or Volatility of the Volatility we consider a GJR specification.

We thus estimate the following model:

$$h_t = \alpha + \delta_s I_s(h_{t-1})h_{t-1} + \beta_d h_{t-1} + \beta_w h_{(t-1:t-5)} + \beta_m h_{(t-1:t-22)} + \gamma_R R_{t-1} + \gamma_{RI} I(R_{t-1})R_{t-1} + \sqrt{\sigma_t} \varepsilon_t$$

$$\sigma_t = \omega + \beta_1 \sigma_{t-1} + \alpha_1 u_{t-1}^2 + \phi_1 u_{t-1}^2 I(u_{t-1})$$

$$\varepsilon_t | \Omega_{t-1} \sim d(0, 1)$$

where h_t is the $\log RRV_{scaled,t}$, $h_{(t-1:t-j)}$ is the HAR component defined as

$$h_{(t-1:t-j)} = \frac{1}{j} \sum_{k=1}^j h_{t-k}$$

with $j = 5$ and 22 in order to capture the weekly and monthly component. $I_s(h_{t-1})$ is an indicator for $RRV_{scaled,t-1}$ bigger than the mean over $s = 5, 10, 22, 44$ and 66 previous days and the unconditional mean (um). These variables capture the asymmetric effects with respect to the volatility. $R = \ln(p_t^{cl}/p_{t-1}^{cl})$ is the return, with p^{cl} the closure price for the day t and $I(R_{t-1})$ is an indicator for negative returns in $t-1$, that captures the asymmetric effects with regard to the lagged return. $u_t = \sqrt{\sigma_t} \varepsilon_t$ is the error term. The full specification for σ_t is a GJR-GARCH to account for the asymmetric effect in the volatility of the volatility, where $I(u_{t-1})$ is an indicator for $u_{t-1} < 0$. Finally, we have 21 model specifications for the mean equation, three variance equations and two distributions for the residuals. In total, 126 models are considered. The estimation and forecast analysis is carried out for different horizons.

4 Estimation and forecast results

We start estimating the model for the entire sample from January 2003 to December 2008. The objective is to assess the impact and significance of our different variables in our models. Then, we compute one-day-ahead out-of-sample rolling forecasting from January 3, 2006 to December 30, 2007 for a total of 502 periods. We have estimated the models until December 30, 2005 and we re-estimate each model at each recursion. To evaluate the performance, we compute the Root mean square error (RMSE) and the Mean absolute error (MAE). We compare the different performances of the models with the Diebold Mariano Test. Table 1 presents the result for the 2003- 2008 estimation, whereas Table 2 presents the forecast performance evaluation.

Estimation results for the full sample period (2003-2008) suggest that HAR components are significant for the three variance specifications and two different distributions. The asymmetric effects with respect to the return and the volatility improve the goodness of fit of the model. The first one is highly significant and it increases the volatility after a negative return. When considering the full specification in the mean equation, the asymmetric effects with respect to the volatility, in the different horizons, are not significant. The asymmetric effect on the previous five days is marginally significant for some models. The sign and significance of the coefficients in the mean equation remain stable for the different specifications in the

Table 1 Estimation results for the 2003-2008

	Model II				Model IX				Model XIV			
	Norm dist Cons Var	GJR-Garch	NIG dist Cons Var	GARCH	Norm dist Cons Var	GARCH	NIG dist Cons Var	GJR-Garch	Norm dist Cons Var	GARCH	NIG dist Cons Var	GJR-Garch
α	-0.332 *** (0.081)	-0.456 *** (0.107)	-0.429 *** (0.082)	-0.456 *** (0.091)	-0.546 *** (0.091)	-0.612 *** (0.100)	-0.570 *** (0.089)	-0.577 *** (0.090)	-0.538 *** (0.092)	-0.607 *** (0.102)	-0.569 *** (0.089)	-0.575 *** (0.090)
β_d	0.364 *** (0.027)	0.348 *** (0.035)	0.333 *** (0.025)	0.328 *** (0.031)	0.322 *** (0.028)	0.316 *** (0.035)	0.304 *** (0.026)	0.313 *** (0.032)	0.279 *** (0.034)	0.271 *** (0.045)	0.274 *** (0.033)	0.290 *** (0.043)
β_u	0.457 *** (0.048)	0.474 *** (0.053)	0.442 *** (0.041)	0.466 *** (0.046)	0.470 *** (0.047)	0.488 *** (0.053)	0.454 *** (0.040)	0.472 *** (0.045)	0.520 *** (0.055)	0.538 *** (0.064)	0.487 *** (0.048)	0.496 *** (0.056)
β_w	0.112 *** (0.041)	0.085 * (0.044)	0.138 *** (0.034)	0.114 *** (0.037)	0.105 *** (0.041)	0.080 * (0.043)	0.133 *** (0.034)	0.105 *** (0.036)	0.104 *** (0.041)	0.079 * (0.043)	0.133 *** (0.034)	0.105 *** (0.036)
δ_5	-	-	-	-	-	-	-	-0.010 (0.006)	-0.009 (0.006)	-0.007 (0.006)	-0.004 (0.006)	-0.004 (0.006)
η_{HET}	-	-	-	-	-10.818 *** (1.364)	-9.679 *** (1.737)	-9.912 *** (1.628)	-9.680 *** (1.807)	-10.821 *** (1.357)	-9.463 *** (1.742)	-9.881 *** (1.629)	-9.653 *** (1.809)
ω	0.197 *** (0.005)	0.027 *** (0.007)	0.193 *** (0.009)	0.024 ** (0.011)	0.192 *** (0.005)	0.023 *** (0.007)	0.188 *** (0.009)	0.031 ** (0.013)	0.192 *** (0.005)	0.022 *** (0.007)	0.188 *** (0.009)	0.030 ** (0.013)
β_1	-	0.819 *** (0.048)	-	0.815 *** (0.073)	-	0.834 *** (0.048)	-	0.788 *** (0.086)	-	0.840 *** (0.047)	-	0.797 *** (0.084)
α_1	-	0.075 *** (0.019)	-	0.057 *** (0.022)	-	0.047 *** (0.014)	-	0.081 ** (0.032)	-	0.046 *** (0.013)	-	0.077 ** (0.031)
ϕ_1	-	-0.072 *** (0.019)	-	-	-	-	-	-0.080 ** (0.038)	-	-	-	-0.076 ** (0.037)
α_{NIG}	-	-	1.579 *** (0.211)	1.735 *** (0.244)	-	-	1.516 *** (0.193)	1.686 *** (0.229)	-	-	1.526 *** (0.197)	1.685 *** (0.229)
β_{NIG}	-	-	0.457 *** (0.137)	0.529 *** (0.157)	-	-	0.391 *** (0.121)	0.483 *** (0.145)	-	-	0.397 *** (0.124)	0.480 *** (0.145)
LLF	-862.8	-838.7	-799.0	-785.2	-845.3	-830.2	-784.1	-769.5	-844.1	-829.3	-783.5	-769.2
AIC	1735.6	1693.3	1612.0	1588.4	1702.6	1676.5	1584.2	1561.0	1702.2	1676.5	1585.0	1562.5
BIC	1761.9	1735.4	1648.8	1635.8	1734.2	1718.5	1626.3	1618.8	1739.0	1723.9	1632.4	1625.6
LJ_{30}	0.504	0.708	0.363	0.622	0.586	0.758	0.594	0.754	0.527	0.711	0.556	0.733
LJ_{40}	0.733	0.839	0.553	0.786	0.830	0.909	0.816	0.905	0.795	0.887	0.791	0.895
$JB-t$	0.001	0.001	-	-	0.001	0.001	-	-	0.001	0.001	-	-
$KS-t$	0.000	0.001	-	-	0.000	0.001	-	-	0.000	0.001	-	-
$LL-t$	0.001	0.001	-	-	0.001	0.001	-	-	0.001	0.001	-	-

Note: Estimation results for the whole sample from January 2003 to December 2008. In this short version, we only present some of the results. LLF is the Log-likelihood function, AIC is the Akaike Information Criteria and BIC is the Bayesian information criterion. Standard errors in bracket. LJ_{30} and LJ_{40} are the Ljung Box test for 30 and 40 lags. $JB-t$ is the Jarque-Bera test for Normality, $KS-t$ is the Kolmogorov-Smirnov and $LL-t$ is the Lilliefors test. "***", "**" and "*" indicates significance at the 10%, 5% and 1%.

variance equation. The inclusion of the GARCH and GJR specification improve the fitting of the models. The models that best fit the series are the ones that include the HAR and leverage effects and GARCH and GJR specifications. Diagnostic check test for the residuals presents different results. Only the models that include the HAR components cannot reject the null hypothesis of no serial correlation in the residual, implying a good performance. Normality Tests for the residuals are rejected for all the models, which is an argument to introduce a non Gaussian distribution. As we said, the results for the models with NIG distribution are similar for the models with Normal distribution. The estimated parameters of the NIG distribution captured the right skewness and excess of kurtosis displayed in the residuals.

For the 2006 forecast, the model that performs better is the autoregressive with HAR components, symmetry over the return, constant variance and NIG distribution. Other models that include asymmetric effects with respect to the volatility over the 5 previous days and the returns perform similarly. Models with different specifications for the variance do not perform better than models with constant variance (measured by the MAE and RMSE). Although, GARCH and GJR improve the goodness of fitness in the estimation, they do not have impact in the forecast. Finally, the Diebold-Mariano tests suggest that models with symmetric effects with respect to the volatility and the returns perform as HAR models. For the 2007, the model that performs better is an HAR component with symmetry over the return and the volatility. The same performance is observed with different variance specification

Table 2 Out-of-sample forecast evaluation for 2006 and 2007, one-day-ahead rolling forecast

Models	MAE	RMSE	Forecast 2006											
			I-	II-	IX-	XIV-	I-	II-	IX-	XIV-	I-	II-	VII-	XIV-
			NIG-C	NIG-C	NIG-C	NIG-C	NIG-GA	NIG-GA	NIG-GA	NIG-GA	NOR-GJ	NOR-GJ	NOR-GJ	NOR-GJ
I-NIG-C	0.331	0.178	-	-	-	-	-	-	-	-	-	-	-	-
II-NIG-C	0.312	0.164	3.25	-	-	-	-	-	-	-	-	-	-	-
IX-NIG-C	0.310	0.164	3.57	0.84	-	-	-	-	-	-	-	-	-	-
XIV-NIG-C	0.311	0.164	3.47	0.34	-0.64	-	-	-	-	-	-	-	-	-
I-NIG-GA	0.331	0.179	-0.56	-3.28	-3.62	-3.52	-	-	-	-	-	-	-	-
II-NIG-GA	0.313	0.165	3.16	-1.40	-1.26	-0.68	3.22	-	-	-	-	-	-	-
IX-NIG-GA	0.311	0.164	3.35	0.27	-1.83	-0.16	3.43	0.69	-	-	-	-	-	-
XIV-NIG-GA	0.312	0.164	3.31	0.04	-1.30	-1.38	3.37	0.39	-0.42	-	-	-	-	-
I-NOR-GJ	0.333	0.180	-0.97	-3.36	-3.73	-3.62	-1.19	-3.34	-3.58	-3.51	-	-	-	-
II-NOR-GJ	0.314	0.165	2.92	-1.11	-1.45	-0.90	3.00	-0.85	-1.04	-0.69	3.19	-	-	-
VII-NOR-GJ	0.312	0.167	2.91	-0.11	-1.03	-0.46	3.01	0.16	-0.57	-0.21	3.21	0.45	-	-
XIV-NOR-GJ	0.312	0.163	3.14	0.13	-0.67	-0.36	3.22	0.44	-0.13	0.23	3.39	0.75	0.27	-
Models	MAE	RMSE	Forecast 2007											
			I-	II-	VII-	VIII-	I-	II-	VII-	VIII-	I-	II-	VII-	VIII-
			NIG-C	NIG-C	NIG-C	NIG-C	NOR-C	NOR-C	NOR-C	NOR-C	NOR-GA	NOR-GA	NOR-GA	NOR-GA
I-NIG-C	0.376	0.259	-	-	-	-	-	-	-	-	-	-	-	-
II-NIG-C	0.365	0.249	0.92	-	-	-	-	-	-	-	-	-	-	-
VII-NIG-C	0.362	0.247	1.16	1.18	-	-	-	-	-	-	-	-	-	-
VIII-NIG-C	0.364	0.246	1.05	0.76	-0.84	-	-	-	-	-	-	-	-	-
I-NOR-C	0.376	0.259	1.15	-0.90	-1.14	-1.03	-	-	-	-	-	-	-	-
II-NOR-C	0.364	0.248	1.02	1.57	-0.62	-0.23	1.00	-	-	-	-	-	-	-
VII-NOR-C	0.361	0.247	1.25	1.67	1.41	1.67	1.24	1.11	-	-	-	-	-	-
VIII-NOR-C	0.363	0.246	1.14	1.16	-0.11	1.54	1.12	0.65	-1.00	-	-	-	-	-
I-NOR-GA	0.376	0.260	1.73	-0.87	-1.10	-0.99	0.68	-0.96	-1.20	-1.08	-	-	-	-
II-NOR-GA	0.365	0.249	0.96	0.66	-0.91	-0.52	0.95	-1.22	-1.44	-0.96	0.91	-	-	-
VII-NOR-GA	0.362	0.247	1.20	1.35	0.39	1.00	1.18	0.80	-1.46	0.34	1.14	1.15	-	-
VIII-NOR-GA	0.363	0.246	1.11	1.01	-0.36	0.72	1.09	0.48	-1.27	-0.82	1.05	0.83	-0.69	-

Note: Forecast performance for 2006 and 2007, 251 observation for each year. Model *I* is an $AR(1)$ specification, *II* is an $AR(1) + HAR$, *VII* is an $AR(1) + HAR + I_{lum}(h_{t-1})h_{t-1} + R_{t-1} + I(R_{t-1})R_{t-1}$, *VIII* is an $AR(1) + HAR + R_{t-1} + I(R_{t-1})R_{t-1}$, *IX* is an $AR(1) + HAR + I(R_{t-1})R_{t-1}$ and *XIV* is an $AR(1) + HAR + I_5(h_{t-1})h_{t-1} + I(R_{t-1})R_{t-1}$. *NIG* indicates Normal Inverse Gaussian distribution, *NOR* is Normal distribution and *C* is a constant variance specification, *GA* is a *GARCH* and *GJ* is a *GJR* variance specification. *MAE* is the Mean Absolute Error. *RMSE* is the Root Mean Square Error. *DMtest* is the DieboldMariano test for equal predictive accuracy between two models based on the MAE. Under H_0 , both models have the same performance. T-statistic in the table. In this short version, we only present some of the results.

and both considered distribution. In contrast with the previous results, for the 2007 forecast, the Diebold Mariano test suggest that there is no evidence to reject the hypothesis of same accuracy between models with HAR components and symmetric effects and the simple $AR(1)$.

5 Conclusions and future steps

In this paper, we have modeled and forecasted price variation through the *Realized Range Volatility* introduced by Martens and van Dijk (2007) and Christensen and Podolskij (2007). We have estimated the series for different sampling frequencies and corrected them with the scaling procedure of Martens and Van Dijk. After the corrections, the volatility stabilized across different sampling frequencies and scaling factors which suggest that the bias caused by the microstructure friction was removed, restoring the efficiency of the estimator. We have considered a model with long memory and asymmetric effects with respect to the return and the volatility in the mean equation and a GARCH and GJR specification for our variance equation, which models the volatility of the volatility. A non Gaussian distribution was also considered for the errors. Results suggest that the HAR models with the asymmetric effect with respect to the volatility are the ones that better fit the model. The analysis of the forecast performance of the different models presented diverse re-

sults. The introduction of asymmetric effects improves the performance. However, there is no evidence to state that these models perform better than the simple HAR while, as we expected, models with GARCH and GJR-GARCH specification do not lead to more accurate previsions than models with constant variance. Future steps in this paper are the possible correction for jumps in the volatility series and the introduction of other financial and macroeconomics variables and days of the week and announcement effects may result in a more accurate prevision.

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