

Long Memory and Nonstationarity in Time Perception Models

Nuno Crato, Ana Diniz, and João Barreiros

Abstract Experimentalists have found stationary long non-periodic waves in human controlled rhythmic movements. For these type of data, namely for tapping tasks, the now-classical Wing-Kristofferson model offers a decomposition of the inter-response intervals based on the sum of two independent short-memory processes: a cognitive component and a motor component. Based on human experiments, we suggest that the cognitive component should be modeled as a long-memory process and the motor component as a white noise process. Our modified W-K model adjusts well to second order properties of observed data. Some probabilistic explanations for long-range dependency have been proposed such as the aggregation of short-memory processes. We propose a new approach which provides insights on the origin of long memory based on previous work on regime-switching processes. Under some assumptions, we obtained the autocorrelation function and the spectral density function of the model. Furthermore, we propose an estimator based on the frequency-domain representation of the likelihood function.

Key words: Time series analysis, long-memory models, $1/f$ noise

1 Introduction

Repetitive human movements exhibit inherent variability reflecting limits on voluntary timing. In his seminal experimental research on this topic, Stevens

Nuno Crato

Cemapre and ISEG, Institute for Economics and Management, R. do Quelhas 6, 1200 Lisboa, Portugal, e-mail: ncrato@iseg.utl.pt

Ana Diniz and João Barreiros

FMH, Faculty of Human Kinetics, Estrada da Costa 1495-688 Cruz Quebrada, Portugal, e-mail: adiniz@fmh.utl.pt

(1886) suggested that variability increases with interval duration. He also proposed two factors to explain this variation—long-term fluctuations as a possible consequence of cognitive processes and short-term fluctuations related to motor limitations. Later, the basic structure of two components for repetitive movements was formally described by Wing and Kristofferson (1973), who offered a model frequently investigated from that time on. Their approach is based on two components—the timekeeper delay or central clock and the motor delay—considered as independent white noise sources. They based their proposal on experiments with short unpaced phases (tapping without a pacing sequence), namely no more than 50 continuous taps. In sequences with this extent, the time series often look stationary, fluctuating randomly around means close to the target values.

Recent experiments with long unpaced phases have revealed that fluctuations typical of long-memory processes appear in repetitive tapping series (Delignières, Lemoine, & Torre, 2004; Wagenmakers, Farrell, & Ratcliff, 2004). There is evidence that long-memory processes occur in other biological series (Chen, Ding, & Kelso, 1997) and in domains as diverse as economics (Granger, 1980), hydrology (Hurst, 1951), or genetics (Voss, 1992). In a previous work (Diniz, Barreiros, & Crato, 2010) we have shown compelling evidence for the presence of long memory in long tapping tasks.

Experiments with long time series may introduce fatigue and drift effects. In fact, the drifting effect toward certain preferred frequencies is expected to be more visible in longer time series and with longer tapping intervals (Ogden & Collier, 1999; Collier & Ogden, 2004). However, careful statistical analysis of tapping data seem to indicate a persistent behavior that can be appropriately modeled as a long memory process of the so-called ARFIMA type (Beran, 1994). This process provides a parsimonious model for these complex data.

In this paper we illustrate these findings with experimental data. Then, we provide an explanation for the phenomenon. Specifically, we provide a stochastic model that both generates the observed behavior and opens a way of shading light into the neurological origins of the observed persistence.

2 The Data

Data originally collected, presented, and discussed at Diniz, Barreiros, & Crato (2010) for long tapping records shows unmistakable signs of stochastic persistence as we can see both visually from the plot of the inter-tap interval durations and the sample autocorrelation function.

There are two striking features on these data. Firstly, level seems to wonder and data look nonstationary. Secondly, when looking at a long time series such as the ones we observed, with more than a thousand data points, departures from the the original level seem to be corrected and data are bound within

reasonable limits. It is not surprising that researchers working with shorter data could have treated it as nonstationary.

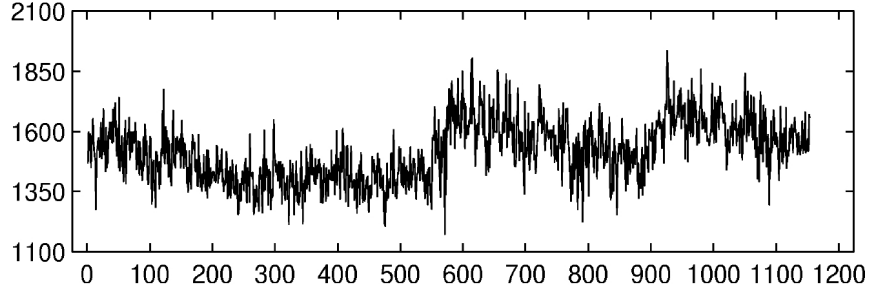


Fig. 1 This figure shows the time intervals maintained by one of some students who volunteered for this study. It shows almost 1200 time intervals obtained after a period of synchronization with a metronome with the target interval of 1.6 seconds. There are local drifts, but the volunteers were able to obtain an overall remarkable timing.

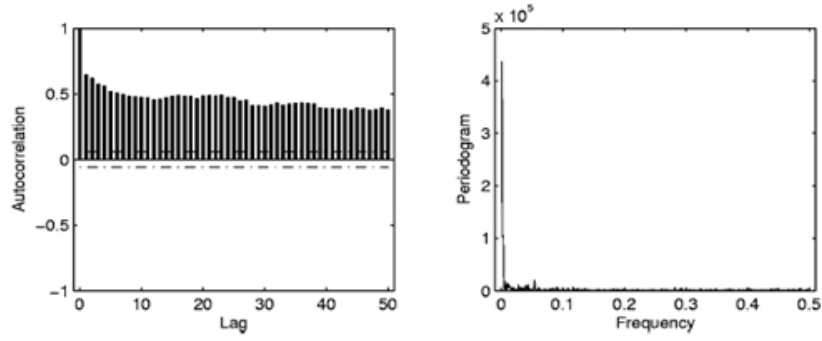


Fig. 2 The autocorrelation function computed for the above data has a notoriously slow decay starting with moderate values. The periodogram shows a peak at short frequencies. These two results are a clear indication of a persistent stationary time series.

3 A Modified Wing-Kristofferson Model

Following pioneering work of Wing and Kristofferson (1973), researchers have dealt with these data considering the separate effects of a cognition and a motor component on the interval times,

$$I_t = C_t + M_t - M_{t-1}, \quad t \in \mathbb{N},$$

that were originally considered as two independent white noise processes. As we have seen, this specification collides with long data records. We are lead to propose a fractional integrated model for the cognitive element, maintaining the white noise model for the motor delay element. This long-memory model with added noise is found in stochastic volatility modeling and its parameters are notoriously difficult to estimate (Crato & Ray, 2002). In its simplest form, the fractional integrated model is simply a fractional noise, which is persistent provided the integration parameter d is not zero. We will have the following specifications

$$C_t := (1 - B)^{-d} Z_t \text{ with } Z_t \sim \text{WN}(0, \sigma_C^2),$$

$$M_t \sim \text{WN}(0, \sigma_M^2).$$

It is straightforward to conclude that this process will have the following autocorrelation function

$$\gamma(k) = \begin{cases} \frac{\Gamma(1-2d)}{\Gamma^2(1-d)} \sigma_C^2 + 2\sigma_M^2, & k = 0, \\ \frac{(-1)\Gamma(1-2d)}{\Gamma(2-d)\Gamma(-d)} \sigma_C^2 - \sigma_M^2, & |k| = 1, \\ \frac{(-1)^{|k|}\Gamma(1-2d)}{\Gamma(|k|-d+1)\Gamma(1-|k|-d)} \sigma_C^2, & |k| \geq 2, \end{cases}$$

and the following spectral function

$$f_I(\lambda) = \left| 1 - e^{-i\lambda} \right|^{-2d} \frac{\sigma_C^2}{2\pi} + \left| 1 - e^{-i\lambda} \right|^2 \frac{\sigma_M^2}{2\pi}, \quad |\lambda| \leq \pi.$$

4 A New Model

We suggest a basic mechanism for the rise of the long-memory behavior. It is inspired on the work of Liu (2000), with regime-switching processes and heavy-tail durations. In our case, the cognitive process is considered to be a regime-switching process and the motor process is considered to be a white noise. The two processes are considered to be mutually independent. To simplify the presentation, we assume that the process $\{I_t\}$ is corrected for the mean μ_I . The process $\{C_t\}$ is now considered as a regime-switching process $\{W_t\}$ with added noise $\{Z_t\}$:

$$\{C_t : C_t = W_t + Z_t\}, \quad \{W_t\} \sim \text{RS}, \quad \{Z_t\} \sim \text{WN}(0, 1),$$

The process $\{W_t\}$ is given by

$$W_t = \sum_{k=1}^{\infty} L_k I(S_{k-1} < t \leq S_k), \quad t \in \mathbb{N}$$

and

$$T_k = S_k - S_{k-1}, \quad k \in \mathbb{N},$$

where $\{T_k\}$ represent the durations (in number of taps) and $\{L_k\}$ represent the levels of latent interval time (in clock time). This way, the regime-switching process can be seen as the successive adoption of cognitive strategies. Let the sequences of durations $\{T_k\}$ and of latent time levels $\{L_k\}$ satisfy

- (i) $P(T_k \in \mathbb{N}) = 1, \quad E(T_k) = \mu_T < \infty, \quad k \in \mathbb{N};$
- (ii) $P(L_k \in \mathbb{R}) = 1, \quad E(L_k) = \mu_L = 0, \quad E(L_k^2) = \sigma_L^2 < \infty, \quad k \in \mathbb{N};$
- (iii) $\{T_k, k \in \mathbb{N}\}$ and $\{L_k, k \in \mathbb{N}\}$ are independent.

Further assume that the variables $\{T_k\}$ have a heavy-tail distribution such that

$$P(T > t) = P(T \geq t+1) = (t+1)^{-\alpha}, \quad t \in \mathbb{N}, \quad 1 < \alpha < 2.$$

Then, the process of duration levels $\{W_t\}$ has the following autocovariance function

$$\gamma_W(k) \sim \frac{\sigma_L^2}{\mu_T(\alpha-1)} k^{1-\alpha}, \quad k \rightarrow \infty, \quad 1 < \alpha < 2.$$

And so the process $\{W_t\}$ has long memory with parameter $d = 1 - \alpha/2$.

With these assumptions we have derived the autocorrelation and the spectral functions of the resulting interval durations

$$\gamma(k) = \begin{cases} \sigma_L^2 + 1 + 2\sigma_M^2, & k = 0, \\ \frac{\zeta(\alpha) - 1}{\zeta(\alpha)} \sigma_L^2 - \sigma_M^2, & |k| = 1, \\ \frac{\zeta(\alpha) - \sum_{l=1}^{|k|} t^{-\alpha}}{\zeta(\alpha)} \sigma_L^2, & |k| \geq 2, \end{cases}$$

$$f_I(\lambda) = \frac{(1 - e^{-i\lambda})^{-1}}{\zeta(\alpha)} \left[e^{-i\lambda} Li_\alpha(e^{i\lambda}) - Li_\alpha(e^{-i\lambda}) \right] \frac{\sigma_L^2}{2\pi} + \frac{1}{2\pi} + \left| 1 - e^{-i\lambda} \right|^2 \frac{\sigma_M^2}{2\pi}, \quad |\lambda| \leq \pi.$$

where we find the Riemann zeta function and the polylogarithm (or Jonquière's) function, respectively,

$$\zeta(\alpha) = \sum_{k=1}^{\infty} \frac{1}{k^\alpha}, \quad Li_\alpha(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^\alpha}.$$

5 Conclusions

Series of intervals for repetitive tapping tasks have been extensively studied in the statistical and human motor control literature. These series serve as a prototype for the study of human cognitive control of time and so can help illuminate brain processes that are important for many human activities. In a very recent paper (Diniz, Barreiros, & Crato, 2010), we have shown that for long data series a simple fractional integrated model works much better than the now classic Wing-Kristofferson model. This is of statistical relevance, but comes short of shading light on the neurological processes involved. In this work, we advance a regime switching process that generates the observed long-memory behavior and is able to provide an explanation for its existence. This is certainly of interest both for the probabilistic and statistical study of long-memory processes and for the study of the neurologic roots of the persistent memory found on brain time-keeping mechanisms.

References

- Beran, J.: Statistics for long-memory processes. New York: Chapman & Hall (1994)
- Chen, Y., Ding, M., & Kelso, J.A.S.: Long memory processes ($1/f^\alpha$ type) in human coordination. *Physical Review Letters* 79, 4501–4504 (1997)
- Collier, G.L., & Ogden, R.T.: Adding drift to the decomposition of simple isochronous tapping: an extension of the Wing-Kristofferson model. *Journal of Experimental Psychology: Human Perception and Performance* 30, 853–872 (2004)
- Crato, N., & Ray, B.K.: Semi-parametric smoothing estimators for long-memory processes with added noise. *Journal of Statistical Planning and Inference* 105, 283–297 (2002)
- Delignières, D., Lemoine, L., & Torre, K.: Time intervals production in tapping and oscillatory motion. *Human Movement Science* 23, 87–103 (2004)
- Diniz, A., Barreiros, J., & Crato, N.: Parameterized estimation of long-range correlation and variance components in human serial interval production. *Motor Control* 14, 26–43 (2010)
- Granger, C.W.J.: Long memory relationships and the aggregation of dynamic models. *Journal of Econometrics* 14, 227–238 (1980)
- Hurst, H.E.: Long-term storage capacity of reservoirs. *Transactions of the American Society of Civil Engineers* 116, 778–808 (1951)
- Liu, M.: Modeling long memory in stock market volatility. *Journal of Econometrics* 99, 139–171 (2000)

- Ogden, R.T., & Collier, G.L.: On detecting and modeling deterministic drift in long run sequences of tapping data. *Communications in Statistics Theory and Methods* 28, 977–987 (1999)
- Stevens, L.T.: On the time sense. *Mind* 11, 393–404 (1886)
- Voss, R.F.: Evolution of long-range fractal correlations and $1/f$ noise in DNA base sequences. *Physical Review Letters* 68, 3805–3808 (1992)
- Wagenmakers, E.J., Farrell, S., & Ratcliff, R.: Estimation and interpretation of $1/f^\alpha$ noise in human cognition. *Psychonomic Bulletin & Review* 11, 579–615 (2004)
- Wing, A. M., & Kristofferson, A. B.: Responses delays and the timing of discrete motor responses. *Perception and Psychophysics* 14, 5–12 (1973)