

Improved estimators of a sensitive proportion through randomized response technique

Giancarlo Diana and Marlies Ranieri

Abstract In this paper, we introduce a class of improved estimators of the proportion of people bearing a sensitive attribute through a randomized response technique. The suggested class allows for the use of auxiliary information (if any) at the estimation stage and for the improvement in efficiency due to the effect of a multiplying constant. Since the optimal estimator depends on the unknown optimal value of this constant, it needs to be estimated. A numerical illustration is also provided to investigate the effect of the improvement on some estimators present in literature. It is shown that the class works well when the sensitive attribute is rare in nature and the population is elusive.

Key words: Auxiliary information; efficiency; optimum estimator; sample design and estimation.

1 Introduction

Direct survey methods often fail to yield reliable data on embarrassing or prohibited acts such as abortion, alcoholism, sexual orientation, drug taking and tax evasion. Social stigma and fear of reprisals result in answering refusal or false reporting, thus affecting estimates by unavoidable bias. [19] first introduced some ingenious procedures, known as *randomized response (RR) techniques*, to estimate the proportion π_A of individuals belonging to a class A , considered socially stigmatizing or inadmissible. Initially, the RR techniques were designed to handle dichotomous (“yes” or “no”) responses to sensitive questions such as “Have you ever used drugs?”. The procedure assumed that two “yes”/“no” questions were provided for each respondent during a face-to-face interview and that a randomization device (RD), like a deck of cards or a dice, was used to establish which question would be answered.

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Since the interviewer would not know the outcome of the randomization mechanism, respondents would perceive that their privacy was protected.

To achieve more confidentiality, [4] developed the unrelated question (UQ) model involving two questions, one related with the sensitive attribute while the other totally innocuous. These pioneering works paved the way for a multitude of models, all aiming at achieving more and more efficiency in the estimation phase. Some of these techniques have been reviewed by [5] and [16].

Up to now, not too much attention has been given to the use of auxiliary information to improve the performance in RR sampling. The ratio method was applied to Warner's procedure by [20], who proposed an estimator which acts as a special case of the class introduced by [3] to estimate a sensitive proportion involving an auxiliary variable with known mean.

Following [12], higher efficiency can be obtained adopting an improved estimator which depends on some multiplying constant whose value is to be suitably chosen by the researcher. [15] applied this approach to improve the estimator of a sensitive proportion under the UQ model. Similarly, [10] improved the estimator given by [8]. The aim of this paper is to provide a general class of improved estimators of the proportion of a sensitive characteristic by enlarging the family of estimators proposed by [3]. The eventuality of including auxiliary information, randomized or not, is comprised as well.

The paper is organized as follows. In Sect. 2, we briefly introduce notation and generalize the class of estimators given by [3] to the case in which also the auxiliary information may be randomized. In Sect. 3, a class of improved estimators is proposed and the exact mean square error (MSE) is given. The best estimator of the suggested class is identified, as well as its estimated version and the minimum attainable MSE. In Sect. 4 we provide a numerical illustration. Finally, concluding remarks are traced in Sect. 5.

2 A class of estimators of a sensitive proportion

Let $U = \{1, \dots, N\}$ be a finite population of N individuals. We denote with Y a variable taking value $Y_i = 1$ if the i -th person bears the sensitive attribute A and $Y_i = 0$ otherwise, $i = 1, \dots, N$. The goal is to estimate the proportion $\pi_A = E(Y)$ of the sensitive attribute A .

Motivated by the work of [3], let us consider a generic RD which allows for the randomization of the responses both on the sensitive question and on the auxiliary information. Let X be a non-sensitive auxiliary variable, apparently neutral but correlated with Y , and assume that its mean, $\bar{X} = N^{-1} \sum_{i=1}^N x_i$, is known. For instance, on studying tax evasion we may survey the type of cars, which is non-sensitive but related with the people's living standards.

We introduce now two dummy variables, S and R , the latter with known mean. S takes value $S_i = 1$ if the i -th person bears a generic attribute B and $S_i = 0$ otherwise. To be explicit, B may be A^c , the complement of A , leading to a Warner's based

method (see [19]), or an innocuous characteristic, implying an UQ based method (see e.g. [4],[8],[17]). On the other hand, R may be interpreted as a function of X , say $f(X)$. The specification of $f(\cdot)$ depends on the codification; numerous options are present in literature (see e.g. [11]). In particular if the identity function is chosen, no randomization is provided (see e.g. [20]).

A sample of n individuals is selected from the population using SRSWR and each respondent is asked to perform a Bernoulli trial with probability of success p , fixed by the researcher. If a success occurs the respondent provides a “yes” (or “no”) response if he/she possesses A and discloses his/her value of X . In case of failure the respondent provides a “yes” (or “no”) response if he/she possesses B , and discloses his/her value of R . The procedure may be summarized as:

$$(Z, V) = \begin{cases} (Y, X), & p \\ (S, R), & 1 - p. \end{cases}$$

Referring to the scheme above, Z is the random variable taking value 1 or 0 according to a “yes” or “no” response. The unknown population proportion of “yes” responses is indicated with $\theta = N^{-1} \sum_{i=1}^N z_i$. Finally, V is the variable that takes the auxiliary values, eventually codified, with known mean $\bar{V} = N^{-1} \sum_{i=1}^N v_i$. Given the SRSWR $\{(z_1, v_1), (z_2, v_2), \dots, (z_n, v_n)\}$, a general class of estimators of π_A may be defined as

$$\hat{\pi}_g = \frac{\hat{\theta}_d - c}{h}, \quad (c > 0, h > 0) \quad (1)$$

where $\hat{\theta}_d = \hat{\theta} - b(\bar{v} - \bar{V})$ is the *difference estimator* (see e.g. [18]) of θ where $\bar{v} = n^{-1} \sum_{i=1}^n v_i$, and $\hat{\theta} = n^{-1} \sum_{i=1}^n z_i = n_1/n$, being n_1 the number of “yes” in the sample responses. We notice that $\bar{v}$ and $\hat{\theta}$ are unbiased estimators, respectively, of $\bar{V}$ and θ . Real constants c and h depends on the adopted RD, while the choice of the real constant b concerns mainly the efficient use of the auxiliary variable. If $b = 0$, no auxiliary information is used at the estimation stage and thus, for fixed c and h , the class reduces to a well specific estimator (with $\hat{\theta}_d = \hat{\theta}$) generated by the adopted RR method. By suitably specifying b , the traditional *ratio*, *product* and *regression* estimators of θ are obtained. Anyhow, *ceteris paribus* no further improvement upon the regression estimator is possible, at least at the first order of approximation. Therefore, formulation (1) includes both classical estimators without supplementary information and possible estimators based on the auxiliary variable, randomized or not. Some estimators proposed in literature belonging to the class are put in evidence in grey in Table 1.

If b is known and c and h satisfy the constraint $\theta = c + h\pi_A$, then $\hat{\pi}_g$ is *unbiased* with variance given by

$$V(\hat{\pi}_g) = \frac{\theta(1-\theta)}{nh^2} + \frac{b}{nh^2}(b\sigma_v^2 - 2\sigma_{vz}), \quad (2)$$

where $\sigma_v^2 = N^{-1} \sum_{i=1}^N (v_i - \bar{V})^2$ and $\sigma_{vz} = N^{-1} \sum_{i=1}^N (v_i - \bar{V})(z_i - \theta)$. The variance (2) can be unbiasedly estimated using

$$\widehat{V}(\hat{\pi}_g) = \frac{\hat{\theta}(1 - \hat{\theta})}{(n-1)h^2} + \frac{b}{nh^2}(bs_v^2 - 2s_{vz}), \quad (3)$$

where $s_v^2 = (n-1)^{-1} \sum_{i=1}^n (v_i - \bar{v})^2$ and $s_{vz} = (n-1)^{-1} \sum_{i=1}^n (v_i - \bar{v})(z_i - \hat{\theta})$. We observe that (2) depends on the constant b . Among several possible values for b , the optimal choice is the one that minimizes (2) itself. It is easy to prove that $V(\hat{\pi}_g)$ attains its minimum when $b_{\text{opt}} = \sigma_{vz}/\sigma_v^2$, i.e. the population linear regression coefficient of Z on V . Replacing b_{opt} in (1), we obtain the optimum estimator in the class, say $\hat{\pi}_{g,\text{opt}}$, which attains the minimum variance bound of the class

$$V(\hat{\pi}_{g,\text{opt}})_{\min} = (1 - \rho_{vz}^2)V(\hat{\pi}_A) = (1 - \rho_{vz}^2) \frac{\theta(1 - \theta)}{nh^2},$$

where ρ_{vz} is the correlation coefficient between Z and V , and $\hat{\pi}_A$ is the analogous estimator defined without taking into account any auxiliary information. Since the non-negative quantity $1 - \rho_{vz}^2$ is the reduction of the variance due to the use of the auxiliary variable, $\hat{\pi}_{g,\text{opt}}$ turns out to be at least as efficient as $\hat{\pi}_A$.

The minimum variance bound can be estimated with

$$\widehat{V}(\hat{\pi}_{g,\text{opt}}) = \frac{\hat{\theta}(1 - \hat{\theta})}{(n-1)h^2} \left(1 - \frac{s_{vz}}{s_z s_v} \right). \quad (4)$$

3 The improved class of estimators

Following [12], a class of improved estimators of π_A may be obtained as

$$\hat{\pi}_{\lambda_g} = \lambda \hat{\pi}_g, \quad (5)$$

where λ is a suitably chosen constant such that $0 < \lambda \leq 1$.

The flexibility of formulation (5) consists on including a multitude of estimators, from the classical till the more elaborate, depending on the researcher's choice to exploit the auxiliary information and/or the improvement.

We investigate now the bias and the MSE of the introduced class. These are, respectively, given by

$$B(\hat{\pi}_{\lambda_g}) = (\lambda - 1)\pi_A$$

and

$$\text{MSE}(\hat{\pi}_{\lambda_g}) = \lambda^2(\pi_A^2 + V(\hat{\pi}_g)) - 2\lambda\pi_A^2 + \pi_A^2. \quad (6)$$

We have that $\text{MSE}(\hat{\pi}_{\lambda_g}) < V(\hat{\pi}_g)$ if

$$0 < \frac{\pi_A^2 - V(\hat{\pi}_g)}{\pi_A^2 + V(\hat{\pi}_g)} < \lambda \leq 1.$$

Table 1 Some estimators included in the class $\hat{\pi}_{\lambda g}$.

Method	Estimator	b	c	h	λ
Warner (1965)	$\hat{\pi}_W$	0	$1 - p$	$2p - 1$	$(0, 1)$
Greenberg et al. (1969)	$\hat{\pi}_G$	0	$(1 - p)\pi_B$	p	$(0, 1)$
Mangat and Singh (1990) ^a	$\hat{\pi}_{MS1}$	0	$p_1 + q_1 q_2$	$q_1 q_2$	$(0, 1)$
Mangat et al. (1992)	$\hat{\pi}_M$	0	$(1 - p)\pi_B$	$1 - (1 - p)\pi_B$	$(0, 1)$
Mangat and Singh (1995) ^b	$\hat{\pi}_{MS2}$	0	p_2	$p_1 - p_2$	$(0, 1)$
Bhargava and Singh (2000) ^c	$\hat{\pi}_{BS}$	0	$p_2 + p_3$	$p_1 - p_2$	$(0, 1)$
Huang (2005)	$\hat{\pi}_H$	0	$1/2$	$1/2$	$(0, 1)$
Shabbir and Gupta (2005) ^d	$\hat{\pi}_{SG}$	0	p_2	$p_1 - p_2 + p_3$	$(0, 1)$
Singh et al. (2003) ^e	$\hat{\pi}_S$	0	$p_2 \pi_B$	p_1	$(0, 1)$
Zaizai (2006)	$\hat{\pi}_Z$	$\hat{\theta}/\bar{x}$	$1 - p$	$2p - 1$	$(0, 1)$
Diana and Perri (2007)	$\hat{\pi}_{DP}$	$(2p - 1)\sigma_{xy}/\sigma_x^2$	$1 - p$	$2p - 1$	$(0, 1)$

a: a first RD provides the sensitive question with probability $1 - q_1 = p_1$, otherwise the respondent passes to a second RD and answers with probability $1 - q_2 = p_2$ to the sensitive question.

b, c, d, e: the RD provides three questions, one sensitive and two innocuous, with probability p_1 , p_2 and $p_3 = 1 - p_1 - p_2$, respectively.

Possible specifications of the proposed class are reported in the whole Table 1. Varying λ in the interval $(0, 1)$, it is possible to improve all the considered estimators. In particular, [15] and [10] applied this approach to improve, respectively, $\hat{\pi}_G$ and $\hat{\pi}_M$.

3.0.1 Optimum estimator in the class $\hat{\pi}_{\lambda g}$

Since λ takes values in $(0, 1)$, $\hat{\pi}_{\lambda g}$ enlarges the class $\hat{\pi}_g$ including also biased estimators. However, we show here that this effect implies at the same time more gains in efficiency. To obtain the optimum estimator, we first minimize (6) with respect to b and further minimize with respect to λ . The solution of the minimization problem leads to the optimum value

$$\lambda_{\text{opt}} = \frac{\pi_A^2}{V(\hat{\pi}_{g,\text{opt}}) + \pi_A^2},$$

which is obviously not meaningful in practice and needs to be estimated. A possible estimator is given by

$$\hat{\lambda}_{\text{opt}} = \frac{\hat{\pi}_{g,\text{opt}}^2}{\widehat{V}(\hat{\pi}_{g,\text{opt}}) + \hat{\pi}_{g,\text{opt}}^2}, \quad (7)$$

where $\widehat{V}(\hat{\pi}_{g,\text{opt}})$ is defined in (4). Following a theoretical line, the bias and the MSE of the optimum estimator $\hat{\pi}_{\lambda_{\text{opt}}} = \lambda_{\text{opt}} \hat{\pi}_{g,\text{opt}}$ are respectively given by

$$B(\hat{\pi}_{\lambda_{\text{opt}}}) = -\frac{V(\hat{\pi}_{g,\text{opt}})\pi_A}{V(\hat{\pi}_{g,\text{opt}}) + \pi_A^2} \quad \text{and} \quad \text{MSE}(\hat{\pi}_{\lambda_{\text{opt}}}) = \frac{V(\hat{\pi}_{g,\text{opt}})\pi_A^2}{V(\hat{\pi}_{g,\text{opt}}) + \pi_A^2}.$$

The latter expression reveals that the optimum estimator is always more precise than $\hat{\pi}_{g,\text{opt}}$. The estimated version of the optimum estimator is

$$\hat{\pi}_{\lambda_{\text{opt}}} = \frac{\hat{\pi}_{g,\text{opt}}^2}{\widehat{V}(\hat{\pi}_{g,\text{opt}}) + \hat{\pi}_{g,\text{opt}}^2} \hat{\pi}_{g,\text{opt}}, \quad (8)$$

and its bias and MSE are respectively given by

$$B(\hat{\pi}_{\lambda_{\text{opt}}}) = \sum_{n_1=0}^n (\hat{\pi}_{\lambda_{\text{opt}}} - \pi_A) \binom{n}{n_1} \theta^{n_1} (1-\theta)^{n-n_1}, \quad (9)$$

and

$$\text{MSE}(\hat{\pi}_{\lambda_{\text{opt}}}) = \sum_{n_1=0}^n (\hat{\pi}_{\lambda_{\text{opt}}} - \pi_A)^2 \binom{n}{n_1} \theta^{n_1} (1-\theta)^{n-n_1}. \quad (10)$$

For $b \neq b_{\text{opt}}$, the previous expressions are still valid replacing $\hat{\pi}_{g,\text{opt}}$ with $\hat{\pi}_g$ and $\widehat{V}(\hat{\pi}_{g,\text{opt}})$ with $\widehat{V}(\hat{\pi}_g)$, defined in (3). For instance, the optimum estimator now is

$$\hat{\pi}_{\lambda_g} = \hat{\lambda} \hat{\pi}_g = \frac{\hat{\pi}_g^2}{\widehat{V}(\hat{\pi}_g) + \hat{\pi}_g^2} \hat{\pi}_g. \quad (11)$$

4 Efficiency comparison

Following [10], a numerical illustration is provided to investigate the effect of the improvement. We have computed the relative efficiency of the estimators in Table 1 and their improved version (11) with respect to Warner's estimator $\hat{\pi}_W$. The relative efficiency is given by

$$RE(\hat{\pi}_{\lambda_g}, \hat{\pi}_W) = V(\hat{\pi}_W) [\text{MSE}(\hat{\pi}_{\lambda_g})]^{-1},$$

where $\hat{\pi}_{\lambda_g}$ is any improved estimator in Table 1 and $\text{MSE}(\hat{\pi}_{\lambda_g})$ is formulated in (10) replacing $\hat{\pi}_{\lambda_{\text{opt}}}$ with $\hat{\pi}_{\lambda_g}$.

The comparisons have been carried out for different values of n , π_A and p . Two values of π_B have been considered in the methods involving an UQ about an innocuous attribute B . We have assumed that $p_1 = p$, $p_2 = 0.7(1-p_1)$ and $p_3 = 0.3(1-p_1)$ in the procedures whose RD provides three statements to be selected with probability p_1 , p_2 and p_3 . Analogous calculations have been made to compare the estimators without improvement (i.e. $\lambda = 1$). We report in Table 2 a restricted but meaningful extract of our results to trace the following conclusions:

Table 2 Empirical efficiency comparisons.

n	10	20	50	10	20	50	10	20	50	for any n		
	$p = 0.6$			$p = 0.7$			$p = 0.8$			$p = 0.6$	$p = 0.7$	$p = 0.8$
	$\pi_A = 0.05$									$\lambda = 1$		
$\hat{\pi}_{\lambda W}$	1.939	2.035	2.071	1.901	1.986	1.962	1.816	1.882	1.760	1	1	1
$\hat{\pi}_{\lambda MS2}$	5.637	5.942	5.960	3.347	3.508	3.383	2.538	2.696	2.450	2.972	1.819	1.498
$\hat{\pi}_{\lambda SG}$	10.479	10.942	10.717	4.597	4.770	4.491	2.965	3.116	2.780	5.554	2.515	1.758
	$\pi_B = 0.01$											
$\hat{\pi}_{\lambda G}$	146.533	110.915	77.394	38.468	28.937	20.374	15.809	11.844	8.444	66.286	18.230	7.825
$\hat{\pi}_{\lambda M}$	213.355	159.959	119.036	49.256	36.889	27.360	18.299	13.691	10.125	117.850	27.006	9.958
	$\pi_B = 0.03$											
$\hat{\pi}_{\lambda G}$	110.085	83.721	63.029	31.527	23.807	17.467	13.970	10.493	7.627	54.108	15.843	7.174
$\hat{\pi}_{\lambda M}$	175.555	133.543	103.600	42.350	31.999	24.362	16.491	12.396	9.297	102.433	24.230	9.241
	$\pi_A = 0.1$											
$\hat{\pi}_{\lambda W}$	1.924	1.998	1.976	1.840	1.846	1.658	1.683	1.605	1.295	1	1	1
$\hat{\pi}_{\lambda MS2}$	5.400	5.530	5.164	3.066	3.016	2.554	2.223	2.105	1.649	2.905	1.755	1.423
$\hat{\pi}_{\lambda SG}$	9.757	9.711	8.533	4.095	3.925	3.918	2.541	2.363	1.829	5.383	2.405	1.658
	$\pi_B = 0.01$											
$\hat{\pi}_{\lambda G}$	61.061	46.039	35.545	15.892	12.076	9.639	6.716	5.156	4.251	36.599	10.155	4.544
$\hat{\pi}_{\lambda M}$	86.015	68.010	59.475	20.072	15.852	13.860	7.751	6.115	5.345	65.054	15.128	5.821
	$\pi_B = 0.03$											
$\hat{\pi}_{\lambda G}$	52.122	39.925	31.726	14.302	10.981	8.875	6.305	4.870	4.034	32.813	9.445	4.355
$\hat{\pi}_{\lambda M}$	77.797	62.227	54.490	18.585	14.793	12.949	7.355	5.830	5.100	60.338	14.286	5.600

- the ordering of efficiency tends to remain unchanged as the parameters vary and the improvement is involved;
- substantial gains in efficiency due to the improvement are generally achieved when p and n are small. In this case, the best performers result by $\hat{\pi}_{\lambda G}$ and $\hat{\pi}_{\lambda M}$, obtained using UQ based methods;
- as p and n increase, the tendency is to reduce the effect of the improvement. Anyhow, large values of p are likely to be misleading as they may induce reticence in answering truthfully.

The bias of the improved estimators has been calculated replacing $\hat{\pi}_{\lambda_{\text{opt}}}$ with $\hat{\pi}_{\lambda_g}$ in (9). The main idea is that the amount of bias is smaller when $\pi_A = 0.05$, varying in the range $(-0.016, -0.035)$. On the contrary, when $\pi_A = 0.1$ the bias is typically larger, varying in the interval $(-0.016, -0.053)$.

5 Concluding remarks

Extending the class provided by [3], the present work has introduced a class of improved estimators of a sensitive proportion. The flexibility of the suggested class is given by the possibility of using supplementary information, randomized or not, and/or the improvement due to the effect of an unknown multiplying constant λ , to be suitably optimized.

Estimating the optimum value λ_{opt} , a numerical illustration has been carried out to improve the performance of some estimators present in literature. According to the conclusions of [10], the proposed approach seems to be particularly appealing

while dealing with situations in which it is difficult to collect large samples and it is presumed that the sensitive attribute is rare in nature. To obtain a substantial gain in efficiency, an improved UQ based method should be adopted fixing p small. Finally, we observe that more efficiency reduces inevitably the confidentiality of the respondents. Hence, when choosing a particular estimator the researcher should find a proper compromise between the two aspects.

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