

A semiparametric Fay-Herriot model for small area estimation of welfare indicators

Caterina Giusti, Stefano Marchetti, Monica Pratesi and Nicola Salvati

Abstract Available data to compute welfare indicators in Italy come mainly from sample surveys, such as the Survey on Income and Living Conditions (EU-SILC). However, these data can be used to produce accurate estimates only at national or regional level. To obtain estimates referring to smaller unplanned domains small area methodologies can be used. In this work we propose a semiparametric version of the Fay-Herriot model to estimate the mean household income in the Municipalities of the Tuscany Region. Our model captures the relationships between neighbouring small areas through bivariate P-splines. In more details, the variables that enter in the area-level model through an unknown smooth bivariate function are the geographical coordinates of the centroids of the Municipalities, while other covariates enter linearly. In the work we also propose a bootstrap estimator for the mean squared error of semiparametric point estimator.

1 Introduction

The computation of local welfare indicators is hungry of data on local situations. In other words, relevant data to support evidence-based policy making must be referred to local government levels. Too often many of the crucial welfare indicators are defined and statistically significant at national or NUTS-2 level (regional level in

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Italy); moreover, they are not always accompanied by a measure of their statistical accuracy. This may hamper the usefulness of the tools employed and the effectiveness of the interventions. In this work we focus on small area estimation methods to provide statistically sound estimates of welfare indicators for unplanned domains identified by LAU 1 and 2 classifications (Provinces and Municipalities in Italy).

The indicator of interest here is the mean household equivalised income in the 287 Municipalities of the Tuscany Region in Italy, computed using data coming from the EU-SILC survey 2007 and from the Census of the Population 2001. The estimator we propose is a semiparametric version of the Fay and Herriot (1979) model; for the corresponding mean squared error we propose a bootstrap estimator.

In principle, a nonparametric or semiparametric model might have significant advantages compared with parametric approaches when the functional form of the relationship between the variable of interest and the covariates cannot be specified a priori, since erroneous specification of the model can result in biased estimators. Even when a specific functional form appears reasonable, the nonparametric model provides a more robust model alternative that can be useful in the process of model checking and validation (Opsomer et al, 2008). In this work we capture the relationships between neighbouring small areas through a semiparametric model based on P-splines for incorporating spatial proximity effects. The variables that enter in the model through an unknown smooth bivariate function are the geographical coordinates of the centroids of the Municipalities, while other covariates enter linearly.

Note that the model proposed here allows to estimate the indicators of interest together with a measure of their variability also for out of sample areas. These can be really numerous when the interest is in obtaining estimates at Municipality level using available survey information. The methods proposed here can easily be extended to the estimation of other income and non-income welfare indicators. Moreover, they can be used to obtain the estimates also in other Italian Regions, to better investigate not only the dissimilarities of welfare indicators inside one Region, but also the so-called “North-South divide” characterizing the Italian territory.

2 The semiparametric Fay-Herriot model

Let θ be the $m \times 1$ vector of the parameter of inferential interest (small area total y_i , small area mean $\bar{y}_i$ with $i = 1 \dots m$) and assume that the $m \times 1$ vector of the direct estimator $\hat{\theta}$ is available and design unbiased. Denote the corresponding $m \times q$ matrix of the area level auxiliary variables by $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_q)$. The Fay and Herriot (1979) model can be expressed as:

$$\hat{\theta} = \mathbf{X}\alpha + \mathbf{D}\mathbf{u} + \varepsilon. \quad (1)$$

Here $\mathbf{u}$ is $m \times 1$ vector of independent and identically distributed random variables with mean $\mathbf{0}$ and $m \times m$ variance matrix $\Sigma_u = \sigma_u^2 \mathbf{I}_m$, $\mathbf{D}$ is $m \times m$ matrix of known positive constants, ε is the $m \times 1$ vector of independent sampling errors with mean $\mathbf{0}$ and known diagonal variance matrix $\mathbf{R} = \text{diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_m^2)$, and α is the $q \times 1$

vector of regression parameters. The Fay-Herriot model is a general linear mixed model with diagonal covariance structure $\Sigma(\sigma_u^2) = \mathbf{D}\Sigma_u\mathbf{D}^T + \mathbf{R}$.

The Fay-Herriot model produces reliable small area estimates by combining the design model and the regression model and then borrowing strength from other domains. It assumes that the direct survey estimators are linear function of the covariates. When this assumption fails down, the Fay-Herriot model can lead to biased estimators of the small area parameters. A semiparametric specification of the Fay-Herriot model, which allows non linearities in the relationship between $\hat{\theta}$ and the auxiliary variables $\mathbf{X}$, can be obtained by penalized-splines.

A semiparametric model with one covariate x_1 can be written as $\tilde{m}(x_1)$, where the function $\tilde{m}(\cdot)$ is unknown, but assumed to be approximated sufficiently well by the function

$$m(\mathbf{x}_1; \eta, \gamma) = \eta_0 + \eta_1 x_1 + \dots + \eta_p x_1^p + \sum_{k=1}^K \gamma_k (x_1 - \kappa_k)_+^p \quad (2)$$

where $\eta = (\eta_0, \eta_1, \dots, \eta_p)^T$ is the $(p+1) \times 1$ vector of the coefficients of the polynomial function, $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_K)^T$ is the coefficient vector of the truncated polynomial spline basis (P-spline) and p is the degree of the spline $(t)_+^p = t^p$ if $t > 0$ and 0 otherwise. The latter portion of the model allows for handling departures from a p -polynomial t in the structure of the relationship. In this portion κ_k for $k = 1, \dots, K$ is a set of fixed knots and if K is sufficiently large, the class of functions in (2) is very large and can approximate most smooth functions. Details on bases and knots choice can be found in Ruppert et al (2003, Chapters 3 and 5). Since a P-spline model can be viewed as random-effects model (Ruppert et al, 2003; Opsomer et al, 2008), it can be combined with the Fay-Herriot model for obtaining a semiparametric small area estimation framework based on linear mixed model regression. Corresponding of the η and γ vectors, define

$$\mathbf{X}_1 = \begin{bmatrix} 1 & x_{11} & \dots & x_{11}^p \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{1m} & \dots & x_{1m}^p \end{bmatrix}, \mathbf{Z} = \begin{bmatrix} (x_{11} - \kappa_1)_+^p & \dots & (x_{11} - \kappa_K)_+^p \\ \vdots & \ddots & \vdots \\ (x_{1m} - \kappa_1)_+^p & \dots & (x_{1m} - \kappa_K)_+^p \end{bmatrix},$$

the mixed model representation of the semiparametric Fay-Herriot model can be written as

$$\hat{\theta} = \begin{bmatrix} \mathbf{X} \\ \mathbf{X}_1 \end{bmatrix} [\alpha, \eta] + \mathbf{Z}\gamma + \mathbf{D}\mathbf{u} + \varepsilon. \quad (3)$$

The $\mathbf{X}_1$ matrix of the model (2) can be added to the $\mathbf{X}$ fixed effect matrix and the model (3) becomes

$$\hat{\theta} = \mathbf{X}\beta + \mathbf{Z}\gamma + \mathbf{D}\mathbf{u} + \varepsilon, \quad (4)$$

where β is a $(q+p+1) \times 1$ vector of regression coefficients, the γ component can be treated as a $K \times 1$ vector of independent and identically distributed random variables with mean $\mathbf{0}$ and $K \times K$ variance matrix $\Sigma_\gamma = \sigma_\gamma^2 \mathbf{I}_K$. The variance-covariance matrix of the model (4) is $\Sigma(\psi) = \mathbf{Z}\Sigma_\gamma\mathbf{Z}^T + \mathbf{D}\Sigma_u\mathbf{D}^T + \mathbf{R}$ where $\psi = (\sigma_\gamma^2, \sigma_u^2)^T$.

Model-based estimation of the small area parameters can be obtained by using the best linear unbiased prediction (Henderson, 1975):

$$\tilde{\theta}^B(\psi) = \mathbf{X}\tilde{\beta}(\psi) + \Lambda(\psi)[\hat{\theta} - \mathbf{X}\tilde{\beta}(\psi)] \quad (5)$$

with $\Lambda(\psi) = (\mathbf{Z}\Sigma_\gamma\mathbf{Z}^T + \mathbf{D}\Sigma_u\mathbf{D}^T)\Sigma^{-1}(\psi)$ and $\tilde{\beta}(\psi) = (\mathbf{X}^T\Sigma^{-1}(\psi)\mathbf{X})^{-1}\mathbf{X}^T\Sigma^{-1}(\psi)\hat{\theta}$.

Extension to bivariate smoothing can be handled by assuming $\tilde{m}(\mathbf{x}_1, \mathbf{x}_2) = m(\mathbf{x}_1, \mathbf{x}_2; \eta, \gamma)$. This is of central interest in a number of application areas as environment and public health. It has particular relevance when referenced responses need to be converted to maps (Opsomer et al, 2008; Pratesi et al, 2008).

3 Mean squared error estimation

The Mean Squared Error estimator (MSE) of $\tilde{\theta}^B(\psi)$, depending on the variance components $\psi = (\sigma_\gamma^2, \sigma_u^2)^T$, can be expressed as (Rao, 2003):

$$MSE[\tilde{\theta}^B(\psi)] = g_1(\psi) + g_2(\psi) \quad (6)$$

where the first term $g_1(\psi) = \Lambda(\psi)\mathbf{R} = \mathbf{R} - \mathbf{R}\Sigma^{-1}(\psi)\mathbf{R}$ is due to the estimation of random effects and is of order $O(1)$, while the second term $g_2(\psi) = \mathbf{R}\Sigma^{-1}(\psi)\mathbf{X}(\mathbf{X}^T\Sigma^{-1}(\psi)\mathbf{X})^{-1}\mathbf{X}^T\Sigma^{-1}(\psi)\mathbf{R}$ is due to the estimation of β and is of order $O(m^{-1})$ for large m .

The estimator $\tilde{\theta}^B(\psi)$ depends on the unknown variance components σ_γ^2 and σ_u^2 . Replacing the parameters with estimators $\hat{\sigma}_\gamma^2, \hat{\sigma}_u^2$, a two stage estimator $\tilde{\theta}^E(\hat{\psi})$ is

$$\tilde{\theta}^E(\hat{\psi}) = \mathbf{X}\hat{\beta}(\hat{\psi}) + \hat{\Lambda}(\hat{\psi})[\hat{\theta} - \mathbf{X}\hat{\beta}(\hat{\psi})]. \quad (7)$$

Assuming normality of the random effects, σ_γ^2 and σ_u^2 can be estimated both by Maximum Likelihood (ML) and Restricted Maximum Likelihood (REML) procedures (Prasad and Rao, 1990).

Under Fay-Herriot models with diagonal covariance matrix, Prasad and Rao (1990) obtained an approximately unbiased estimator of the MSE. Following the results of Prasad and Rao (1990) and Das et al (2004), Opsomer et al (2008) extended the Prasad-Rao MSE estimator to models with more general covariance structure. An approximately unbiased estimator of the MSE is

$$mse[\tilde{\theta}^E(\hat{\psi})] = g_1(\hat{\psi}) + g_2(\hat{\psi}) + 2g_3(\hat{\psi}). \quad (8)$$

which is the same estimator derived by Prasad and Rao (1990). In formula (8), the term $g_3(\hat{\psi})$ appears twice due to a bias correction of $g_1(\hat{\psi})$.

We propose an alternative procedure for estimating the MSE of the EBLUP $\tilde{\theta}^E(\hat{\psi})$ based on bootstrapping following the bootstrap procedure proposed by González-Manteiga et al (2007) and Opsomer et al (2008). In this procedure,

the bootstrap random effects $(\gamma_1^*, \dots, \gamma_K^*)^T$, $(u_1^*, \dots, u_m^*)^T$ and the random errors $(\varepsilon_1^*, \dots, \varepsilon_m^*)^T$ are obtained by resampling respectively from the empirical distribution of the predicted random elements $\hat{\gamma} = (\hat{\gamma}_1, \dots, \hat{\gamma}_K)^T$, $\hat{\mathbf{u}} = (\hat{u}_1, \dots, \hat{u}_m)^T$, and the residuals $\hat{\mathbf{r}} = \hat{\boldsymbol{\theta}} - \mathbf{X}\hat{\boldsymbol{\beta}} - \mathbf{Z}\hat{\boldsymbol{\gamma}} - \mathbf{D}\hat{\mathbf{u}} = (\hat{r}_1, \dots, \hat{r}_m)^T$, previously standardized. This method avoids the need of distributional assumptions and it is more robust to non-normality of any of the random components of the model with respect to the analytical MSE estimator, as suggested by some preliminary simulation results (Giusti et al, in preparation). The procedure works as follows:

1. Fit model (4) to the initial data $\hat{\boldsymbol{\theta}}$, obtaining estimates $(\hat{\sigma}_\gamma^2, \hat{\sigma}_u^2)$ and $\hat{\boldsymbol{\beta}}$.
2. With estimates obtained in step 1, calculate predictors of $\hat{\gamma} = (\hat{\gamma}_1, \dots, \hat{\gamma}_K)^T$ and $\hat{\mathbf{u}} = (\hat{u}_1, \dots, \hat{u}_m)^T$. Then take $\hat{\gamma}^S = \hat{\Sigma}_\gamma^{-1/2} \hat{\gamma}$ and $\hat{\mathbf{u}}^S = \hat{\Sigma}_u^{-1/2} \hat{\mathbf{u}}$ where $\hat{\Sigma}_\gamma^{-1/2}$ and $\hat{\Sigma}_u^{-1/2}$ are the root square of the generalized inverse of $\hat{\Sigma}_\gamma^{-1/2} = \mathbf{Z}\hat{\Sigma}_\gamma \mathbf{Z}^T \mathbf{P}(\hat{\psi}) \mathbf{Z}^T \hat{\Sigma}_\gamma \mathbf{Z}$ and $\hat{\Sigma}_u^{-1/2} = \mathbf{D}\hat{\Sigma}_u \mathbf{D}^T \mathbf{P}(\hat{\psi}) \mathbf{D}^T \hat{\Sigma}_u \mathbf{D}$, respectively, obtained by the spectral decomposition. It is convenient re-scale the elements $\hat{\gamma}_k^S$ and $\hat{u}_i^S$ so that they have sample means exactly equal to zero and sample variances $\hat{\sigma}_\gamma^2, \hat{\sigma}_u^2$. It is achieved by the transformation

$$\hat{\gamma}_k^{SS} = \frac{\hat{\sigma}_\gamma \left\{ \hat{\gamma}_k^S - K^{-1} \sum_{j=1}^K \hat{\gamma}_j^S \right\}}{\sqrt{K^{-1} \sum_{d=1}^K \left\{ \hat{\gamma}_d^S - K^{-1} \sum_{j=1}^K \hat{\gamma}_j^S \right\}^2}}, \quad k = 1, \dots, K$$

$$\hat{u}_i^{SS} = \frac{\hat{\sigma}_u \left\{ \hat{u}_i^S - m^{-1} \sum_{j=1}^m \hat{u}_j^S \right\}}{\sqrt{m^{-1} \sum_{d=1}^m \left\{ \hat{u}_d^S - m^{-1} \sum_{j=1}^m \hat{u}_j^S \right\}^2}}, \quad i = 1, \dots, m.$$

Construct the vectors $\gamma^* = (\gamma_1^*, \dots, \gamma_K^*)^T$ and $u^* = (u_1^*, \dots, u_m^*)^T$, whose elements are obtained by extracting a simple random sample with replacement of size K and m from the sets $\hat{\gamma}^{SS} = (\hat{\gamma}_1^{SS}, \dots, \hat{\gamma}_K^{SS})^T$ and $\hat{\mathbf{u}}^{SS} = (\hat{u}_1^{SS}, \dots, \hat{u}_m^{SS})^T$, respectively. Then calculate the bootstrap quantity of interest $\boldsymbol{\theta}^* = \mathbf{X}\hat{\boldsymbol{\beta}} + \mathbf{Z}\gamma^* + \mathbf{D}u^* = (\theta_1^*, \dots, \theta_m^*)^T$.

3. Compute the vector of residuals $\hat{\mathbf{r}} = \hat{\boldsymbol{\theta}} - \mathbf{X}\hat{\boldsymbol{\beta}} - \mathbf{Z}\hat{\boldsymbol{\gamma}} - \mathbf{D}\hat{\mathbf{u}} = (\hat{r}_1, \dots, \hat{r}_m)^T$. Standardize the residuals by $\hat{\mathbf{r}}^S = (RP(\hat{\psi})R)^{-1/2} \hat{\mathbf{r}}$. Re-standardized these values

$$\hat{r}_i^{SS} = \frac{\left\{ \hat{r}_i^S - m^{-1} \sum_{j=1}^m \hat{r}_j^S \right\}}{\sqrt{m^{-1} \sum_{d=1}^m \left\{ \hat{r}_d^S - m^{-1} \sum_{j=1}^m \hat{r}_j^S \right\}^2}}, \quad i = 1, \dots, m.$$

Construct the vector $\mathbf{r}^* = (r_1^*, \dots, r_m^*)^T$, whose elements are obtained by extracting a simple random sample with replacement of size m from the set $\hat{\mathbf{r}}^{SS} = (\hat{r}_1^{SS}, \dots, \hat{r}_m^{SS})^T$. Then take $\boldsymbol{\varepsilon}^* = (\varepsilon_1^*, \dots, \varepsilon_m^*)^T$ where $\varepsilon_i^* = \sigma_{ir_i^*}$.

4. Construct bootstrap data from the model $\hat{\boldsymbol{\theta}}^* = \boldsymbol{\theta}^* + \boldsymbol{\varepsilon}^* = \mathbf{X}\hat{\boldsymbol{\beta}} + \mathbf{Z}\gamma^* + \mathbf{D}u^* + \boldsymbol{\varepsilon}^* = (\hat{\theta}_1^*, \dots, \hat{\theta}_m^*)^T$.

5. Regarding $\hat{\beta}$, $\hat{\sigma}_\gamma^2$ and $\hat{\sigma}_u^2$ as the true values of β , σ_γ^2 and σ_u^2 , fit the model (4) to the bootstrap data $\hat{\theta}^*$. The obtained estimates β^* , σ_γ^{2*} and σ_u^{2*} will be called bootstrap estimators.
6. Calculate the bootstrap small area estimator using β^* , σ_γ^{2*} and σ_u^{2*} in place of the ‘true’ $\hat{\beta}$, $\hat{\sigma}_\gamma^2$ and $\hat{\sigma}_u^2$, $\tilde{\theta}^{E*}(\hat{\psi}^*) = X\hat{\beta}^*(\hat{\psi}^*) + \hat{\Lambda}^*(\hat{\psi}^*)[\hat{\theta}^* - X\hat{\beta}^*(\hat{\psi}^*)]$.
7. Repeat steps 2-6 B times. In the b -th bootstrap replication, let $\theta_i^{*(b)}$ be the quantity of interest in area i , $\tilde{\theta}_i^{E*}(\hat{\psi}^{*(b)})$ be the bootstrap estimator for area i .

Our proposed MSE estimate can then be obtained by adding the bootstrap estimate $g_{3i}^{NPB}(\hat{\psi})$ and the analytical estimates $g_{1i}(\hat{\psi})$ and $g_{2i}(\hat{\psi})$, and then including a bootstrap bias correction of $g_{1i}(\hat{\psi}) + g_{2i}(\hat{\psi})$ (Pfeffermann and Tiller, 2006), as

$$mse_i^{bcNPB}[\tilde{\theta}_i^E(\hat{\psi})] = 2[g_{1i}(\hat{\psi}) + g_{2i}(\hat{\psi})] - B^{-1} \sum_{b=1}^B [g_{1i}(\hat{\psi}^{*(b)}) + g_{2i}(\hat{\psi}^{*(b)})] + g_{3i}^{NPB}(\hat{\psi}). \quad (9)$$

$$\text{where } g_{3i}^{NPB}(\hat{\psi}) = B^{-1} \sum_{b=1}^B \left\{ \tilde{\theta}_i^{E*}(\hat{\psi}^{*(b)}) - \tilde{\theta}^{B*}(\hat{\psi}) \right\}^2.$$

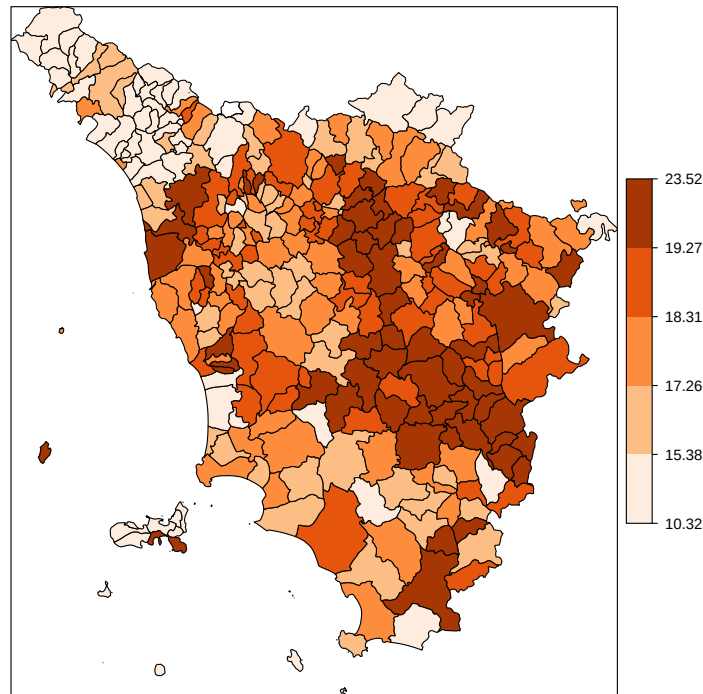
4 Application to the estimation of welfare estimators

In Italy, the European Survey on Income and Living Conditions (EU-SILC) is conducted yearly by ISTAT to produce estimates on the living conditions of the population at national and regional (NUTS-2) levels. Regions are planned domains for which EU-SILC estimates are published, while Provinces (LAU-1 level) and Municipalities (LAU-2) are unplanned domains. It is useful to note that some Provinces may have very few sampled Municipalities; furthermore, many Municipalities are not even included in the sample at all. Direct estimates may therefore have large errors at provincial level or they may not even be computable at municipality level, thereby requiring resort to small area estimation techniques.

In this section we present an application using data from the 2007 EU-SILC survey. This dataset, with data coming from the 2001 Population Census of Italy, represent a valuable source of information to compute welfare estimates in Italy. In the present application our target is the estimation of the mean equivalised household income in the Municipalities of the Tuscany Region. The mean values at Municipality level of the household equivalised income, of some household characteristics (size, ownership of the house, squared meters of the house) and of individual characteristics of the head of the household (occupational status, age, gender, marital status, education level) are available from the EU-SILC survey 2007 for 59 of the 287 Municipalities in the Region; the same covariate information is available from the Census 2001 for each household living in Tuscany. The coordinates (latitude and longitude) of the centroids of the Municipalities are also available, and their enter in the model for the mean income through a bivariate P-spline component.

Figure 1 represent the mean household equivalised income in each of the 287 Municipalities of the Region. The higher estimates are observed for the Municipalities of Firenze, Siena, Arezzo and Lucca, as well as for some of their neighbouring areas. The lower estimates are observed for the Municipalities in the Massa-Carrara Province and for those in the Northern part of the Lucca Province (North-West part of the Region), for some of the coastal and insular areas of the Municipality of Livorno and finally for the Municipalities in the North-East of the Region. These results are consistent with those obtained in previous studies using the same data at provincial level (Giusti et al, 2009). Note however that there is a high variability of the estimates for Municipalities belonging to the same Province; thus, conducting the analysis at provincial level would mask important dissimilarities characterizing those territories. Finally, table 1 shows the good performance of the proposed bootstrap estimator of the mean squared error (9): the 75% of the values of the percentage coefficient of variation estimated for the 287 areas vary between the 2.6% and the 9.5% (minimum and third quartile of the 287 estimated CVs%). These results further allows to investigate the dissimilarities between the areas of interest, underlining the usefulness and potentialities of the proposed methods to obtain significant estimates of welfare indicators at the small area level.

Fig. 1 Mean household equivalised income in Tuscany Municipalities, thousands of Euros.



	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
RMSE	0.4571	1.035	1.269	1.369	1.637	3.989
CV%	2.6320	5.614	7.006	8.124	9.513	28.270

Table 1 Summary for the Root Mean Squared Error (thousands of Euros) and Percentage Coefficient of Variation of the 287 estimates.

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