

Duration-Dependent Markov-Switching VAR Models with Applications to the Business Cycle Analysis

*Modelli Markov-Switching VAR con probabilità di transizione dipendente
dalla durata degli stati e applicazioni all'analisi del ciclo economico*

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Riassunto: Il modello autoregressivo a regimi markoviani di Hamilton viene generalizzato in due direzioni: i) il modello qui presentato consente di modellare serie autoregressive vettoriali (VAR), i cui valori medi dipendono da una comune variabile dicotomica non osservabile, che evolve secondo una catena di Markov, ii) le probabilità di transizione della catena di Markov sono lasciate dipendere dalla durata di ciascuno stato. Il modello è poi applicato alle quattro serie storiche che il NBER utilizza per datare il ciclo economico statunitense, ed i risultati vengono comparati con la datazione NBER del ciclo. Inoltre il modello viene utilizzato per testare la presenza di dipendenza delle probabilità di transizione dalla durata degli stati dell'economia (recessione, espansione).

Keywords: Markov-Switching Model, Hidden-Markov Model, Vector Autoregression, Duration-Dependence, Gibbs Sampling, MCMC, Business Cycle Analysis.

1. Introduction and motivation

Hamilton (1989) introduced the Markov-switching models (from now on MS) to the business cycle researchers. He applied the MS model to the real U.S. GDP and showed how good the probability of being in recession or expansion generated by his model matched the NBER dating. Since then, the studies on business cycles that exploited the MS model have grown exponentially. Nevertheless, the basic MS model, when used to model the business cycle, has at least two theoretical weaknesses: (i) it is univariate, (ii) it does not allow duration dependence. Since business cycles are fluctuations of aggregate economic activity, which express themselves through the comovements of many macro variables, point (i) is not a negligible limitation. The multivariate generalization of the MS model was carried out by Krolzig (1997), in his excellent work on the MS-VAR model. As far as point (ii) is concerned, some authors, such as Diebold and Rudebusch (1990), Diebold *et al.* (1993), Watson (1994), have found evidence of duration dependence in the U.S. business cycles, and therefore, as Diebold *et al.* (1993) point out, the MS model results misspecified.

In the present paper a duration dependent Markov switching VAR model is presented and applied to the four macroeconomic time series that the NBER uses to date the U.S. business cycle. Bayesian inference on the unknown quantities of the model is carried out using Markov chain Monte Carlo methods, which outrun some limitation of the maximum-likelihood approach to such models.

A similar, although univariate, model was used by Kim and Nelson (1999, section 10.3.1) to test duration-dependence in the U.S. business cycle, but their inference on the

state variable was carried out using a *single move Gibbs sampler*, which, in our experience, has a too slow convergence to the invariant distribution to be feasible in such a problem. In the next section it will be shown how to reformulate the inhomogeneous Markov chain used by Kim and Nelson in a homogeneous way, so that the fast converging *multimove Gibbs sampler* can be exploited.

2. The Model

The duration-dependent MS-VAR model is defined as follows:

$$\mathbf{y}_t = \boldsymbol{\mu}_0 + \boldsymbol{\mu}_1 S_t + \mathbf{A}_1(\mathbf{y}_{t-1} - \boldsymbol{\mu}_0 - \boldsymbol{\mu}_1 S_{t-1}) + \dots + \mathbf{A}_p(\mathbf{y}_{t-p} - \boldsymbol{\mu}_0 - \boldsymbol{\mu}_1 S_{t-p}) + \boldsymbol{\varepsilon}_t \quad (1)$$

where $\mathbf{y}_t$ is a vector of observable variables, S_t is a binary (0-1) unobservable random variable following a Markov chain with varying transition probabilities, $\mathbf{A}_1, \dots, \mathbf{A}_p$ are coefficient matrices of a stable VAR process, and $\boldsymbol{\varepsilon}_t$ is a gaussian (vector) white noise with covariance matrix $\boldsymbol{\Sigma}$.

In order to achieve duration dependence for S_t , a Markov chain is built for the pair (S_t, D_t) , where D_t is the duration variable defined as follows:

$$D_t = \begin{cases} D_{t-1} + 1 & \text{if } S_t = S_{t-1} \\ 1 & \text{if } S_t \neq S_{t-1} \end{cases} \quad (2)$$

A maximum value, τ , for the duration variable D_t must be fixed, so that the Markov chain (S_t, D_t) is defined on a finite state space. When $(S_t = s, D_t = \tau)$ holds, only four events are given non-zero probabilities:

$$\begin{aligned} & (S_t = i, D_t = \tau) | (S_{t-1} = i, D_{t-1} = \tau) \quad i = 0, 1 \\ & (S_t = i, D_t = 1) | (S_{t-1} = j, D_{t-1} = \tau) \quad i \neq j, \quad i, j = 0, 1 \end{aligned}$$

with the interpretation that, when the economy has been in state i at least τ times, the additional periods in which the state remains i influence no more the probability of transition.

As pointed out by Hamilton (1994, section 22.4), it is always possible to write the likelihood function of $\mathbf{y}_t$, depending only on the state variable at time t , even though in the model a p -order autoregression is present; this can be done using the extended state variable $S_t^* = (D_t, S_t, S_{t-1}, \dots, S_{t-p})$, which comprehends all the possible combinations of the states of the economy in the last p periods. The transition matrix $\mathbf{P}^*$ of the Markov chain S_t^* will usually be a relatively huge matrix, although rather sparse, having a maximum number 2τ of non-zero elements.

In order to reduce the number (2τ) of non-zero elements in $\mathbf{P}^*$ to be estimated, a more parsimonious probit specification is used. Consider the linear model

$$S_t^\bullet = [\beta_1 + \beta_2 D_{t-1}] S_{t-1} + [\beta_3 + \beta_4 D_{t-1}] (1 - S_{t-1}) + \epsilon_t \quad (3)$$

with $\epsilon_t \sim \mathcal{N}(0, 1)$, and $S_t^\bullet$ latent variable defined by

$$\Pr(S_t^\bullet \geq 0 | S_{t-1}, D_{t-1}) = \Pr(S_t = 1 | S_{t-1}, D_{t-1}) \quad (4)$$

$$\Pr(S_t^\bullet < 0 | S_{t-1}, D_{t-1}) = \Pr(S_t = 0 | S_{t-1}, D_{t-1}). \quad (5)$$

It's easy to show that it holds

$$p_{1|1}(d) = \Pr(S_t = 1 | S_{t-1} = 1, D_{t-1} = d) = 1 - \Phi(-\beta_1 - \beta_2 d) \quad (6)$$

$$p_{0|0}(d) = \Pr(S_t = 0 | S_{t-1} = 0, D_{t-1} = d) = \Phi(-\beta_3 - \beta_4 d) \quad (7)$$

where $d = 1, \dots, \tau$, and $\Phi(\cdot)$ is the standard normal cumulative distribution function. Now the four parameters β completely define the transition matrix $\mathbf{P}^*$.

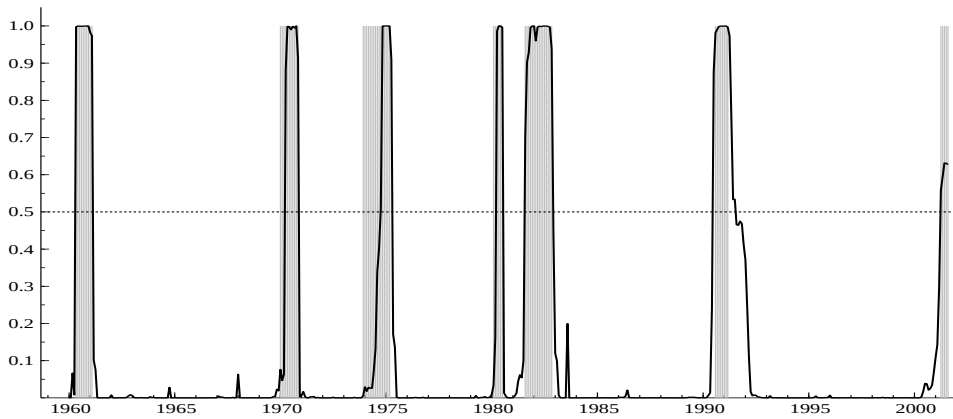
Inference on the unknowns of the model will be carried out using Bayesian MCMC methods, which, due to space restrictions, cannot be discussed here.

3. Application to the U.S. business cycle analysis

The model proposed in the previous section has been applied to the log-difference of the four time series, on which the NBER relies to date the U.S. business cycle: industrial production (IP), total nonfarm-employment (EMP), total manufacturing and trade sales in million of 1996\$ (TRADE), personal income less transfer payments in billions of 1996\$ (INCOME).

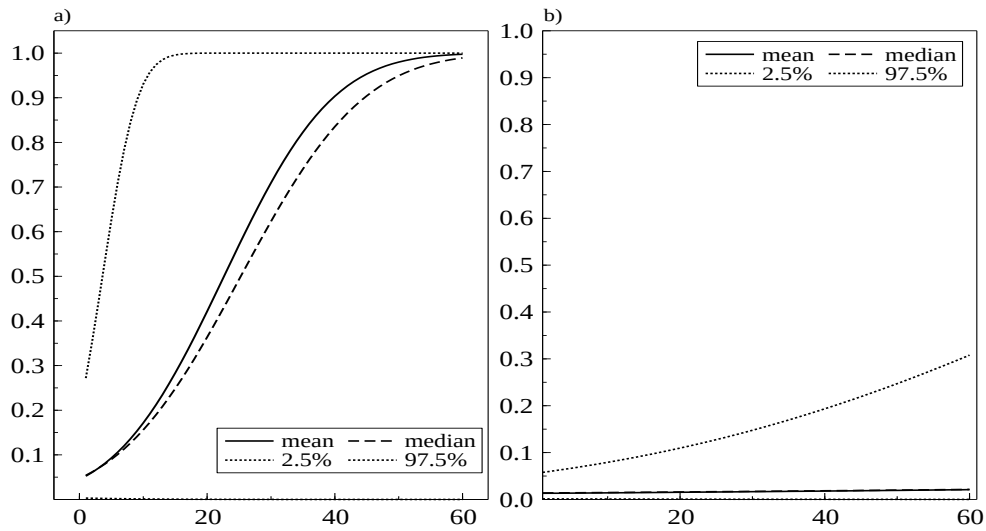
The model, with $\tau = 60$ and $p = 2$ did not work too well, while the results in absence of the VAR part ($p = 0$) are rather encouraging¹. Summaries of the marginal posterior distributions are shown in the table below, while figure 1 compares the probability of the U.S. economy being in recession resulting from the estimated model with the official NBER dating of the U.S. business cycles: the signal “probability of being in recession” extracted by our model matches rather well the official dating, and is less noisy than the signal extracted by Hamilton (1989). Figure 2 shows how the duration of a state (recession or expansion) influences the transition probabilities: while the probability of moving from a recession into an expansion seems to be influenced by the duration of the recession, the probability of falling into a recession appears to be independent of the length of the expansion.

Figure 1: Probability of recession (line) and NBER dating (gray shade)



¹This is probably due to the fact that the duration dependent MS model is a stationary process, which, therefore, can be approximated with an autoregressive model. Thus, the duration dependent switching part and the VAR part try to “explain” almost the same features of the series, and the model is not too well identified.

Figure 2: Probability of moving a) from a recession into an expansion after d months of recession b) from an expansion to a recession after d months of expansion



Parameter	Prior		Posterior				
	mean	var	mean	s.d.	2.5%	50%	97.5%
μ_0 IP	-0.300	1.000	-0.708	0.145	-0.987	-0.707	-0.435
μ_0 EMP	-0.300	1.000	-0.200	0.036	-0.269	-0.200	-0.132
μ_0 TRADE	-0.300	1.000	-0.417	0.132	-0.688	-0.413	-0.167
μ_0 INCOME	-0.300	1.000	-0.111	0.056	-0.224	-0.111	0.004
μ_1 IP	1.500	1.000	1.130	0.146	0.853	1.129	1.410
μ_1 EMP	1.500	1.000	0.440	0.034	0.376	0.439	0.504
μ_1 TRADE	1.500	1.000	0.795	0.139	0.527	0.792	1.076
μ_1 INCOME	1.500	1.000	0.449	0.056	0.341	0.449	0.562
β_1	0.000	5.000	2.224	0.358	1.591	2.203	2.982
β_2	0.000	5.000	-0.003	0.007	-0.018	0.003	0.010
β_3	0.000	5.000	-1.698	0.478	-2.708	-1.672	-0.839
β_4	0.000	5.000	0.075	0.067	-0.032	0.066	0.232

References

- Diebold F. and Rudebusch G. (1990) A nonparametric investigation of duration dependence in the american business cycle, *Journal of Political Economy*, 98, 596–616.
- Diebold F., Rudebusch G. and Sichel D. (1993) Further evidence on business cycle duration dependence, in: *Business Cycles, Indicators and Forecasting*, Stock J. and Watson M., eds., The University of Chicago Press.
- Hamilton J.D. (1989) A new approach to the economic analysis of nonstationary time series and the business cycle, *Econometrica*, 57, 357–384.
- Hamilton J.D. (1994) *Time Series Analysis*, Princeton University Press.
- Kim C.J. and Nelson C.R. (1999) *State-space models with regime switching: classical and Gibbs-sampling approaches with applications*, The MIT Press.
- Krolzig H.M. (1997) *Markov-Switching Vector Autoregressions. Modelling, Statistical Inference and Application to Business Cycle Analysis*, Springer-Verlag.
- Watson J. (1994) Business cycle durations and postwar stabilization of the u.s. economy, *American Economic Review*, 84, 24–46.