

Contemporaneous Aggregation in Seasonally Adjusted Time Series

L'Aggregazione Contemporanea nella Destagionalizzazione delle Serie Temporal

Riccardo Gatto

Istat, via Ravà 150, 00142 Roma,

e-mail: rigatto@istat.it

Filomena Di Rienzo

Università di Roma "la Sapienza"

e-mail: mena_dr@yahoo.com

Riassunto: Nella recente letteratura il problema della aggregazione simultanea di serie storiche destagionalizzate è affrontato solo come ricerca di un criterio di scelta tra metodologia diretta e indiretta. Le risposte sono spesso altamente teoriche e scarsamente applicabili alle procedure reali dei diversi istituti nazionali di statistica. Nuove metodologie sono proposte in questo lavoro e semplici criteri di scelta vengono utilizzati per confrontarle tra loro.

Keywords: Contemporaneous aggregation, Seasonally adjusted time series, Direct and indirect procedures

1. Introduction

The problem of contemporaneous aggregation in seasonally adjusted time series arises from the evidence that no procedure, among those mostly used, is completely linear. This means that if x_t , x_{1t} and x_{2t} are three time series such that $x_t = x_{1t} + x_{2t}$ for each $t \in N$ (where t is time) and $SA(\cdot)$ is a seasonal adjusting procedure then not necessarily $SA(x_t) = SA(x_{1t}) + SA(x_{2t})$. The publication of a large number of series, linked by linear constraints is, on the other side, a very common need for the national statistics institutes. This is the reason behind the recent interest of the econometric literature in the problem of contemporaneous aggregation. The most important recent works approach the problem by reducing it at a criterion of choice between direct or indirect methodology. Direct methodology consists in publishing as final seasonal adjusted series the result of a simple direct application of the procedure, and this for all the series regardless of the inconsistency of the set of series with respect to the constraint. Let $FSA(\cdot)$ be the final seasonal adjusting operator then the direct approach simply states: $FSA(x_t) = SA(x_t)$ and $FSA(x_{it}) = SA(x_{it})$ for $i=1,2$ and for each t . Indirect methodology requires direct application only for the component while the aggregate series is obtained by adding the adjusted components. Then $FSA(x_t) = SA(x_{1t}) + SA(x_{2t})$, while $FSA(x_{it}) = SA(x_{it})$ for $i=1,2$ and for each t . Of course the indirect methodology is fully consistent.

In order to choose between these two approaches some highly theoretical criteria are suggested, very interesting under the methodological point of view but not easily applicable in practice, especially if dealing with a large number of series.

In the next section some alternative methodologies are proposed in addition to the direct and indirect ones. In the third section easy empirical criteria are suggested in order to help in the choice. In the last section classic and alternative methodologies are

compared by means of the criteria introduced in the third section, applying them on the Italian labour force survey series.

2. Alternative Methodologies

Given the necessity for consistency among time series the Italian Statistic Institute (ISTAT) has chosen the pure indirect approach. In the labour force survey press release 16 seasonally adjusted series are presented (*series of interest*). They are linked by complex constraints: since they are published by labour status and geographical area but, regarding the employment, they are also published by economic activity. Implementing the pure indirect approach means applying the adjustment procedure to a set of unpublished series: cross data by area and activity, the so-called *elementary series* set. The elementary series are then added up to the series of interest. This means that almost no series is directly adjusted, with all the problems that can arise by fault of aggregation: loss of information contained in the more aggregated series (the most reliable), spurious seasonality, etc.

The pure indirect approach is not the only one having the full consistency property. Two other approaches are going to be presented and both they have this property.

The first alternative is the so-called *pivot series* methodology. Among the series of interest a sub-set is chosen (pivot series set) such that all other series are obtained as a solution of the system of the linear constraints. Then the adjustment procedure will be applied on the pivot series and the remaining series will be obtained by solving the system. Following the former example it is possible to carry out a very simple application: let $\{x_t ; x_{1t}\}$ be chosen as the pivot series set, then $FSA(x_t)=SA(x_t)$; $FSA(x_{1t})=SA(x_{1t})$ and $FSA(x_{2t})=SA(x_t)-SA(x_{1t})$, for each t . The more complex the system of variables the largest the number of feasible pivot series sets (and the more evident the utility of this method). In the application presented in the fourth section (on Italian LFS series) three different pivot series have been studied, each of them consisting of 9 series. The first set (Pivot Series I) contains the series corresponding to the variables with the largest possible average frequencies over time. The second set (Pivot Series II) contains randomly chosen series. The third set (Pivot Series III) contains the series corresponding to the variables with the smallest possible average frequencies along time. The *pivot series* method allows the publication of the largest possible number of directly adjusted series, ensuring for them the maximum of quality. To be studied the effect on the other series.

The second proposed alternative, to the classic methods, is the so-called *proportionally adjusting* methodology. Series at a certain level of aggregation are seasonal adjusted directly, the series at lower level of aggregation are first directly adjusted and then weighted in order the system of constraints to hold. Series at upper levels of aggregation are obtained adding appropriately up the lower level series. Stated that $FSA(x_t)=SA(x_t)$ a realisation can be: $FSA(x_{it})=SA(x_{it})*SA(x_t)/[SA(x_{1t})+SA(x_{2t})]$ for $i=1,2$ and for each t , where $SA(x_t)/[SA(x_{1t})+SA(x_{2t})]$ is the weight for time t . Other choices can be done with respect to the weighting system. Such kind of models are not new in literature (see den Butter and Fase, 1991) but they are not approached by recent authors, probably because of their very empirical nature, while literature is evolving through theoretical works. Even in this case a complex system of constraints brings several feasible realisations on a same set of series. In the application presented in the fourth section the method named

Proportional Adj. I has been obtained by adjusting directly the two series of national employment and unemployment, then weighting the adjusted series (between the series to be published) at a lower level of aggregation and adding up the results to obtain the series for the labour forces. In the method named *Proportional Adj. II* the series at the lower level of aggregation are not directly seasonally adjusted (before being weighted) but obtained as a sum of the elementary adjusted series.

The *proportionally adjusting* method uses the information, about seasonality, from all the available series, and allocates eventual errors among all the lower level series.

3. Empirical Criteria for the choice

To help in the choice among the several methods and their feasible applications few criteria are needed. Recent theoretical approaches suggest the main problems that can affect the seasonal adjusting indirect procedures. Otranto and Triacca (2000) point their attention on the distance between the models generating the direct and the indirect series. Campolongo and Planas (2000) and Gomez (2000) stress the problem of the revisions (how numbers change, even for the past data, when new information is added over time). Stability of the estimated series is of course a much appreciated feature. A further problem is the arising of spurious seasonality caused by aggregation or by other transformation part of the indirect methods. Here three empirical criteria are suggested to compare different methods. They have been studied in order to be as easy as possible, feasible even when applied to a large number of series. They are all conceived as euclidean distances between vectors where the vectors are two series in the space of time. The method showing the minimum in one of the distances is the best under that criterion.

Following Otranto and Triacca's idea the first distance is between directly and indirectly adjusted series. It gives a measure of the information lost not considering the pure direct approach. For the *i*-th series and for the *j*-th method the distance is then:

Direct vs. indirect distance

$$\sqrt{\sum_t (FSA_j(x_{it}) - SA(x_{it}))^2}$$

Revisions are measured by the distance between a series and itself, calculated with the same method but with a one-year shorter information set, over the first common year of the two series. It gives a measure of the stability over time of the procedure. In case of quarterly data, for the *i*-th series and for the *j*-th method, at time *T* this distance will be:

Revisions measure

$$\sqrt{\sum_{t=4}^{t-7} (FSA_{[0;T]j}(x_{it}) - FSA_{[0;T-4]j}(x_{it}))^2}$$

where the information set is indicated in squared brackets.

The distance between a seasonally adjusted series and itself, after a second application of direct procedures, measures spurious seasonality. For the *i*-th series and for the *j*-th method the distance is then:

Spurious seasonality measure

$$\sqrt{\sum_t (FSA_j(x_t) - SA[FSA_j(x_t)])^2}$$

The distances are applied on all the 16 series of interest and, in order to get synthetic indicators, it is suggested to calculate the average and the maximum value over the 16 series.

4. Comparing several models of aggregation

The several methods presented in the second section have been compared by means of the criteria presented in the third section. They have been applied to the LFS series produced by ISTAT also presented in the second section. The seasonal adjusting procedure used is the one chosen by ISTAT: the TRAMO-SEATS in the WINDOWS version. All the applications have been run in a full automatic way (a setting called RSA=3). The pure indirect method is presented in two versions: the *elementary series* follows the rules already showed, while in the *Istat* method TRAMO-SEATS is not run automatically: every series is modelled with accuracy (This is the ISTAT standard). In Table 1 the results of the experiment are showed. The methods are ordered according to the average distance of direct vs. indirect series.

Table 1: *Title of the table*

METHODS	Direct vs. indirect		Revisions		Spurious seasonality	
	Max	Average	Max	Average	Max	Average
Proportional Adj. I	4,925	1,767	511	73	4,467	1,310
Pivot Series II	16,815	3,007	9,374	771	6,026	2,028
Pivot Series III	18,720	3,755	2,827	242	4,091	924
Pivot Series I	18,720	4,393	8,856	657	6,897	2,391
Proportional Adj. II	13,473	4,688	4,870	428	4,467	1,306
Istat	24,951	6,590	9,489	967	2,313	673
Elementary Series	44,555	11,109	12,755	2,881	7,302	1,570

Evidences shows that there is no “optimum” method, the best under all criteria; that the procedure actually used is the best only under the minimum spurious seasonality criterion while the proportional adjusting methods seem to be an interesting field of research. Results encourage further studies in order to achieve better procedures for dealing with multivariate series, better both from the quality point of view and the practical one.

References

- F.A.G. den Butter, M.M.G. Fase (1991), *Seasonal Adjustment as a Practical Problem*, North-Holland, 147-187.
- Gòmez V. (2000), *Revision-based Test for Direct versus Indirect Seasonal Adjustment of Aggregated Series*, Spanish Ministry of Finance.
- Otranto Edoardo e Triacca Umberto (2000), *A Distance-based Method for the Choice of Direct or Indirect Seasonal Adjustment*, Istituto Nazionale di Statistica, Roma.
- Planas Christophe e Campolongo Francesca (2000), *The Seasonal Adjustment of Contemporaneously Aggregated Series*, Institute for Systems, Informatics and Safety, Joint Research Centre of European Commission.