

Methodological Problems in Index Numbers' Construction For Multi-Temporal Comparison in the Financial Field

Problemi metodologici per la costruzione di numeri indice per il confronto multitemporale in ambito finanziario

Flavio Verrecchia (*), Biancamaria Zavanella (**)

Dipartimento di Statistica, Università degli Studi di Milano Bicocca,

Via Bicocca degli Arcimboldi, 8 - 20126 Milano,

: flavio.verrecchia@unimib.it - biancamaria.zavanella@unimib.it

Riassunto: Gli indici di borsa più diffusi sono tradizionalmente a base fissa e sono costruiti come medie aritmetiche ponderate dei rapporti di prezzo (Costa 1999).

L'obiettivo è di accertare se la formula di Laspeyres è ottimale per effettuare confronti di lungo periodo con cambiamenti molto veloci della struttura del mercato. Quando i prezzi e le quantità possono considerarsi funzioni continue del tempo, la misura più adeguata per sintetizzare le loro variazioni è costituita dall'indice di Divisia. In questo lavoro si confrontano i risultati forniti dalla formula di Laspeyres con quelli delle più note formule della famiglia $P(x,y)$ degli indici assiomaticamente corretti (Paasche, Fisher e Sato-Vartia) al fine di ottenere indicazioni sull'accuratezza delle misure fornite per l'andamento del mercato finanziario, dagli indici (MIB) tradizionalmente calcolati.

Keywords: financial data, continuous index number, Divisia index, chain index number

1. Introduction and aims ()**

The aim of this paper is the discussion of financial markets' price indices, a very particular kind of index numbers which are characterized by the continuous disposability over time of prices and quantities of swapped securities.

The most popular stock exchange indices are fixed-base ones and are built as weighted arithmetic means of prices ratios (Costa (1999)); particularly, the official Milan stock exchange index is calculated through a Laspeyres formula (Mibtel 3 Jan 1994=10000).

This paper discusses the performances of the Laspeyres formula in long run comparisons (Zavanella (1994)), when the market's structure changes very quickly. When prices and quantities can be considered as continuous functions of time, the Divisia index is the most proper synthetic measure of their variation (Forsyth, Fowler (1981), Trivedi (1981), Koves (1983)). Unfortunately, it is necessary to know the exact mathematical form of the continuous functions for calculating the original Divisia's formula, nevertheless it is possible to find good discrete approximations in the chain-indices of $P(x,y)$ family. Particularly, the chain-index $P(1,0)$ (Sato-Vartia) is the formula which more quickly approximates the Divisia index (Zavanella (2000), Martini (2001)). This is an attractive way of solving the financial index problem and it will be the topic of wider research of which this paper is just the beginning.

In order to underline methodological problems linked to index numbers' construction in the financial field, in this first phase of the work, time series of MIB are rebuilt with fixed-base prices indices by using the best known formulas of P(x,y) family, i.e.: Paasche, Fisher and Sato-Vartia. The axiomatic theory of index numbers has been used to choose indices.

The data source used is Datastream, a data bank containing financial and macro-economic information: shares and bonds, reports, market indices, interest and exchange rates, macro-economic data, futures, options and warrants. The daily data from 03/01/94 to 20/02/02 of the securities inside the MIB index have been employed, in particular closing prices and exchanged quantities, where available.

The Data Warehousing (DW) methodology has supported the first phases of the process; especially the access to sources (Back End phase) and the realization of data bases (Datamart) concerning time series of prices and volumes (DW phase).

2. Results (*)

The irremissible axiomatic properties of index numbers and their cofactors are: strong proportionality (PR), homogeneity (H) commensurability (C), monotonicity (M) and associativity (A). If both an index and its cofactor do not satisfy one of these properties they cannot be considered proper measures of prices and quantities variations. It was shown (Martini 1992, 2001) that indices satisfying irremissible axiomatic properties (together with their cofactors) belong to the family of geo-logarithmic indices (P(x,y) family) and their crossing. The ${}_0P_t(x,y)$ family indices are geometric means of n prices ratios¹ weighted with the logarithmic mean² of relative values (w_{txi} , w_{0yi} ; $i=1, 2, \dots, n$) built with the compared situation (t) prices and q_{xi} quantities and the base situation (0) prices and q_{yi} quantities.

Among the infinite indices ($\forall x, y \in [0,1]$) the most significant are: Laspeyres ${}_0P_t(0,0)$, Paasche ${}_0P_t(1,1)$, Fisher and Sato-Vartia ${}_0P_t(1,0)$. Laspeyres and Paasche indices also satisfy the aggregativity³ (G) property, but not base (B) and factor (F) reversibility, while Fisher and Sato-Vartia have F and B but not G.

The Bortkiewicz index, $({}_0P_t(1,1)-{}_0P_t(0,0))/{}_0P_t(0,0) = [s_p/P(0,0)][s_q/Q(0,0)] = s_{pq}$, has the same sign of price and quantity ratios covariance: it is null when covariance s_{pq} or when price, s_p^2 , or quantity, s_q^2 , ratios variances are null. The more Laspeyres and Paasche indices are different the more price and quantity ratios covariance and variance are high. When the Bortkiewicz is negative the security index having a more favourable trend for prices is the Laspeyres (its cofactor $Q(1,1)$), the Paasche index is more unfavourable for

¹ The geo-logarithmic mean of the period compared (t) with the base period (0) is defined by the following expression:

$${}_0P_t(x,y) = \prod_i {}_0P_{ti}^{j_i / \sum_i j_i} \quad \text{with} \quad j_i = \Lambda(w_{txi}, w_{0yi}) ; \begin{cases} q_{xi} = q_{ti}^x \cdot q_{0i}^{1-x} & (0 \leq x \leq 1) \\ q_{yi} = q_{ti}^y \cdot q_{0i}^{1-y} & (0 \leq y \leq 1) \end{cases}$$

² The logarithmic mean of two positive quantities, a and b, is defined by the following expression:

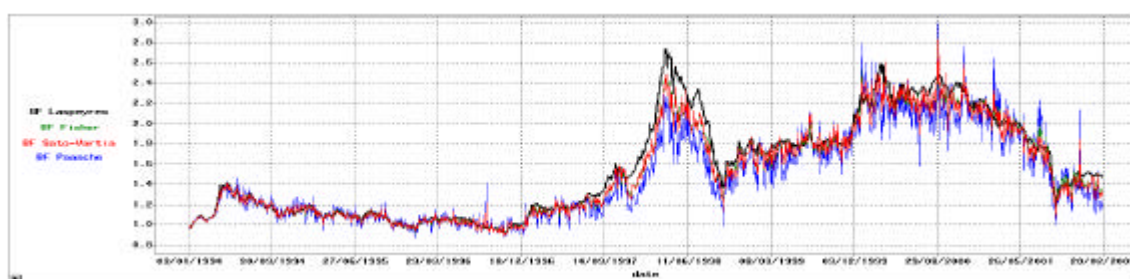
$$\Lambda(a,b) = \begin{cases} (a-b) / (\ln a - \ln b) & \text{for } a \neq b \\ a & \text{for } a = b \end{cases}$$

³ As known they are expressed respectively: Laspeyres as arithmetic mean weighted with the base period (0) structure and Paasche as harmonic mean weighted with the compared period (t) structure.

quantities), while the price Paasche seems less favourable for prices (more favourable is its cofactor, $Q(0,0)$, the quantity Laspeyres). When covariance is not null the Bortkiewicz index measures the variability range of axiomatically correct indices, whose Laspeyres and Paasche are the two bounds of the existence range of axiomatically correct indices. Fisher and Sato-Vartia are intermediate points of the existence range.

The formula's choice is not neutral, on the contrary the deriving information can influence the considered market. The choice of the Paasche or Laspeyres formula can lead to significantly different results, indeed the covariance between relative prices and quantities is high. In particular financial data also allow the build up of bi-characteristic indices. In figure 1. while fixed-base Laspeyres and Paasche indices tend to diverge, bi-characteristic indices Sato-Vartia and Fisher, take very similar values, intermediate between Laspeyres and Paasche (e.g. in date 19/02/02 indices values are: Sato-Vartia 1.28, Fisher 1.31, Laspeyres 1.47 and Paasche 1.17).

Figure 1: Fixed-base price index numbers: Laspeyres, Paasche Fisher and Sato-Vartia



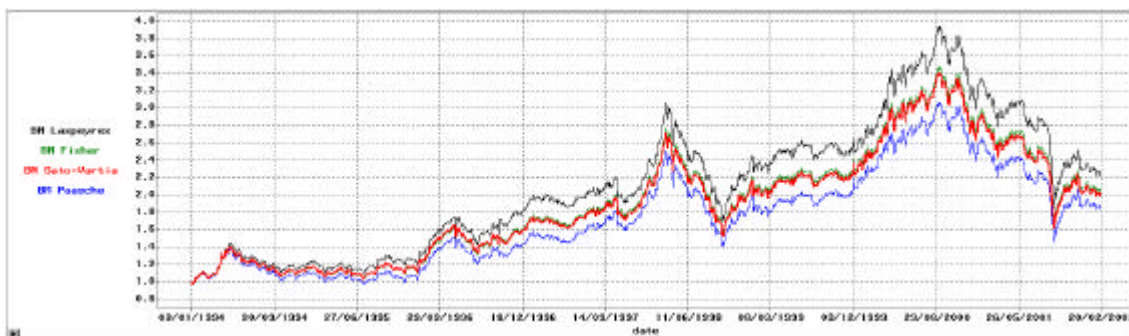
Fixed-base indices are not representative because the securities value common to both situations, decrease over time for example in Table 1; ratio value common to both situations on total value is about 16%.

Table 1: Moving-base (BM) and fixed-base (BF) price index numbers

Var % 19-02-02/18-02-02	BF Var%	BM Var %
Laspeyres	-1.31%	-0.79%
Paasche	-6.38%	-0.86%
Fisher	-3.88%	-0.82%
Sato-Vartia	-4.81%	-0.81%
Securities common to both situations	28	297
Ratio value common to both situations on total value (%)	16.19%	99.97%

Chain price index numbers of ${}_0P_t(x,y)$ family indices is presented in figure 2. In this case too the formula's choice is not neutral. While fixed-base Laspeyres and Paasche indices tend to diverge over time, bi-characteristic indices take very similar values, intermediate between Laspeyres and Paasche. In particular the Fisher index is represented, intermediate between Paasche and Laspeyres but the Sato-Vartia index shows a more stable trend (CV%: Laspeyres 35.25, Fisher 33.35, Sato-Vartia 32.67). Chain price index numbers are generally higher than fixed-base ones, because securities' prices and quantities of base situation over time, grow less than new ones.

Figura 2: Chain price index numbers: Laspeyres, Paasche, Fisher and Sato-Vartia



Conclusions

Until a few years ago choosing the fixed-base and the Laspeyres formula was determined by operative problems. The disadvantage of such a choice is given by the progressive loss of representativeness of securities' base period lists, with the introduction of new ones in the Stock exchange. Through the new available tools (e.g.: DW) it is possible, today, to compute moving-base indices by using comparable price and quantity data, referring to securities in both compared periods. Advantages of using the moving-base are the following:

- 1) the computed index ($P(x,y)$ formula) is more representative than the fixed-base index because it is built on a greater number of securities;
- 2) among the different formulas the moving-base Sato-Vartia is to be preferred as it approximates the Divisia index, when the time interval dividing the two situations decreases.

Finally, with the DW it is possible to compute axiomatically correct indices, which are good approximations of the Divisia index, at shorter intervals (today the MIB is computed every minute).

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