

The Cointegration Analysis in Hypothesis of Heteroschedasticity with the Wild Bootstrap

L'Analisi di Cointegrazione in Ipotesi di Eteroschedasticità mediante il Bootstrap Esterno

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Riassunto: Il concetto di cointegrazione (Engle, Granger, 1987) ha assunto sempre maggiore importanza in questi ultimi anni ed è stato recentemente oggetto di interessanti estensioni all'ambito dei processi frazionari (cointegrazione frazionaria). Spesso si parla di cointegrazione in riferimento a serie macroeconomiche e finanziarie, le quali, tuttavia, risultano frequentemente affette da eteroschedasticità. Tale aspetto è di estremo rilievo se si considera che l'omoschedasticità è una delle condizioni sottostanti l'applicazione delle principali procedure proposte in letteratura per testare la cointegrazione. L'obiettivo di questo lavoro è quello di valutare nell'ambito dell'analisi di cointegrazione, tramite simulazioni, in ipotesi di eteroschedasticità, le performance di un test Bootstrap Esterno e di confrontarlo con alcuni test noti in letteratura.

Keywords: Cointegration, Wild Bootstrap

1. Introduction

Testing for the existence of a cointegration relationship received much attention in literature. The main works are, for example, those of Engle and Granger 1987, Johansen 1988, 1996, Phillips and Durlauf, 1986.

Most of the results about the estimators for non-stationary series are asymptotic and their goodness has been evaluated by Montecarlo experiments, for example Schwert 1989, Diebold and Rudenbush, 1991. If the samples are finite, it seemed that the most frequently used tests exhibit serious level of size distortion and different powers.

These results oriented the most recent researches about this theme to the development of new procedures of estimation and testing by using the bootstrap method. In this context, it is interesting the proposal of a test for unit root which employs the bootstrap method called Wild Bootstrap or External Bootstrap, Procidano and Rigatti Luchini, 1999, 2000. This test seems to be robust in hypothesis of heteroschedasticity, in particular, in case of GARCH errors.

In this work we want to verify if the External Bootstrap test is robust as regards heteroschedasticity also in the context of the cointegration analysis, Gerolimetto and Procidano, 2001. To do that, we compare, by simulations, the performances of some procedures for the cointegration analysis: the Dickey-Fuller test (DF), Dickey and Fuller, 1979, the Sargan-Bhargava test (DW), Sargan, Bhargava, 1983, Bhargava, 1986,

the Johansen tests (λ_{Trace} and λ_{max}), Johansen, 1988 to the performances of the External Bootstrap test (B) in hypothesis of GARCH errors, typical of several economical and financial time series. In this context, like in the unit root analysis (Procidano and Rigatti Luchini, 1999, 2000, 2001), the simulations show a robustness of the External Bootstrap test as regards heteroschedasticity, particularly for small samples. These results could push the expansion of the researches to the fractional cointegration context, considering that fractional modelling significantly expands the possibilities of cointegration analysis and poses new challenges.

2. The Simulation Experiment

In this work the most important procedures for cointegration analysis are compared. In particular, we consider the Johansen tests λ_{Trace} and λ_{max} , and, in the context of the “two steps procedures”, the Dickey-Fuller test, the Sargan-Barghava test and the External Bootstrap test.

The External Bootstrap test employs the resampling procedure called External bootstrap or Wild bootstrap proposed by Wu in 1986. It consists of carrying out extractions from an external distribution with zero mean and unit variance and it is robust as regards heteroschedasticity.

The comparison has been realized by the simulation of 5000 time series of length $T=25, 50, 100, 200$ ¹ for each of the two following schemes, partially referring to what Lee and Tse did in their work in 1996:

1) Non cointegrated systems with GARCH (1,1) errors, i.e.:

$$\nabla X_t = \varepsilon_t$$

2) Cointegrated systems GARCH (1,1) errors, i.e.

$$\nabla x_{1,t} = -0.2(x_{1,t-1} - x_{2,t-1}) + \varepsilon_{1,t}$$

$$\nabla x_{2,t} = \varepsilon_{2,t}$$

Where $X_t = (X_{1t}, X_{2t})$ is a vector of 2 integrated time series and $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})$ is the error term vector whose components are assumed to be normally distributed with zero mean and variance:

$$\sigma_{it}^2 = \phi_{i0} + \phi_{i1} \varepsilon_{i,t-1}^2 + \phi_{i2} \sigma_{i,t-1}^2$$

In the first case, the simulations permit to evaluate the size distortion of the different tests of cointegration expressed as proportion of rejects of the null hypothesis, in the second case the power of the different tests is estimated.

¹ The sample sizes $T=25$ and $T=50$ have been chosen accordingly with other previous analogous works e.g. Banerjee, Dolado, Galbraith, Hendry, 1993 (pagg. 214-218).

The parameters of the GARCH model have been chosen like in Procidano and Rigatti Luchini, 1999, 2000, 2001, Lee and Tse, 1996, i.e. considering quasi-integrated ($\phi_1 + \phi_2 \cong 1$) and quasi-degenerate ($\phi_0 \cong 0$) structures.

3. Results

In the following table the size distortions of the tests are reported.

Table1 *Proportions of rejects out of 5000 tests, Nom. Lev. 0.95
500 replications, normal error distribution.*

Test	T=25	T=50	T=100	T=200
$\phi_0 = 0.1 \quad \phi_1 = 0.3 \quad \phi_2 = 0.6$				
DF	0.0668	0.0628	0.0621	0.0593
DW	0.0715	0.0678	0.0631	0.0618
λ_{Trace}	0.0701	0.0691	0.0591	0.0572
B	0.0598	0.0583	0.0569	0.0521
λ_{max}	0.0737	0.0652	0.0636	0.0609
$\phi_0 = 0.05 \quad \phi_1 = 0.3 \quad \phi_2 = 0.65$				
DF	0.0684	0.0678	0.0659	0.0612
DW	0.0721	0.0682	0.0628	0.0624
λ_{Trace}	0.0798	0.0672	0.0622	0.0604
B	0.0619	0.0614	0.0593	0.0554
λ_{max}	0.0816	0.0705	0.0624	0.0607
$\phi_0 = 0.001 \quad \phi_1 = 0.3 \quad \phi_2 = 0.699$				
DF	0.0692	0.0689	0.0678	0.0627
DW	0.0734	0.0712	0.0695	0.0653
λ_{Trace}	0.0786	0.0688	0.0646	0.0617
B	0.0563	0.0638	0.0621	0.0612
λ_{max}	0.0833	0.0677	0.0636	0.0621

$$\phi_0 = \sigma_0^2(1 - \phi_1 - \phi_2), \sigma_0^2 = 1$$

From the analysis of the table, it is possible to see how the size distortion of all the tests tends to approach the nominal level as the sample size increases. Moreover, the size distortion exhibited by the B test is always the lowest for every sample size and for every parametric choice. This result is particularly important for small sample sizes (e.g. T=25, T=50) where the test B permits to get proportions of rejects closer to the nominal level than those got by the other tests. As the sample size increases the difference becomes less evident because the size distortion tends to approach the nominal level.

As far as the power is concerned (we do not report the table, but it is available by the authors), even though the power of the B test is not always the best, anyway it turns out to be high and compared to the DF test both compared to the other test in general. For every test the power increases as the sample size increases.

These results seem to confirm the robustness of the External Bootstrap test with respect to heteroschedasticity also in the interesting context of the cointegration analysis. In our opinion, these are very important results, particularly because they could be profitably employed in the recent and challenges extensions of cointegration concept to fractional modelling (fractional cointegration).

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