

Expanding the Information Set in Smooth Transition GARCH Models ¹

Insieme Informativo Allargato nei Modelli GARCH a Transizione Liscia

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Riassunto: Nel contesto della volatilità dei mercati finanziari assumono un ruolo chiave i modelli della classe GARCH. Tra le varie estensioni proposte nel corso degli anni, prenderemo in considerazione i modelli GARCH a transizione liscia, la cui peculiarità è di ammettere la presenza di più regimi di volatilità, tra i quali si alterna una diversa specificazione GARCH sulla base di una qualche funzione continua che determina il passaggio da un regime all’altro. Scopo del lavoro è valutare il comportamento di tali modelli quando variabili esogene sono inserite nell’equazione della varianza condizionata.

Keywords: Switching volatility, GARCH models, smooth transition, persistence.

1. Introduction and econometric framework

Financial markets volatility is one of the most prominent area of research in finance. Over the years, a number of different features about returns volatility have emerged, such as volatility clustering, high persistence, mean reverting and nonlinearity. The most widely used class of models to estimate and forecast volatility is represented by the GARCH processes (Bollerslev *et al.* (1994)), which are able to capture these stylized facts and consider the conditional variance of returns as measure of volatility. In this paper we paid our attention to a less-known class of GARCH models, introduced independently by Hagerud (1997) and González-Rivera (1998), and developed by Anderson *et al.* (1999). Intuitively, the basic idea of the model is to consider different states of the world or regimes (for example, high and low volatility) and to allow for the possibility that the future behavior of the conditional variance depends on the current regime. As consequence, a different model will be active in different states and the regime-switching behavior is determined by a continuous function which arises to a dynamical pattern similar to that introduced by Teräsvirta (1994) in the STAR models for the conditional mean equation. Following Bollerslev *et al.* (1994), a GARCH class of models can be expressed as

$$\epsilon_t | I_{t-1} \sim N(0, \sigma_t^2) \quad (1)$$

where I_{t-1} denotes the information set and σ_t^2 is a time-varying variance. Different specifications of σ_t^2 arise to several kinds of GARCH-type models, each of them include different features of the returns volatility. In particular, Anderson *et al.* (1999) consider a very flexible parametrization of σ_t^2 which includes these stylized facts and allows to conditional variance to change smoothly between different regimes. Formally, a resulting

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Asymmetric Nonlinear Smooth Transition GARCH (ANST-GARCH) is

$$\sigma_t^2 = \omega_1 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + (\omega_2 + \alpha_2 \epsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2) \times F(\epsilon_{t-1}; \gamma) + \psi X_t \quad (2)$$

where $F(\epsilon_{t-1}; \gamma)$ indicates the logistic function,

$$F(\epsilon_{t-1}; \gamma) = (1 + \exp\{-\gamma \epsilon_{t-1}\})^{-1}, \gamma > 0 \quad (3)$$

which is bounded between 0 and 1, and depends on the transition variable ϵ_{t-1} and on the smoothness parameter γ restricted to be strictly positive in order to avoid identification problems. Note that for $\gamma = 0$, $F(\epsilon_{t-1}; \gamma) = 1/2$ arising to a one state, or linear GARCH model and when $\gamma \rightarrow \infty$, $F(\epsilon_{t-1}; \gamma)$ becomes a step-function indicating an abrupt change between a two regimes, as in the GJR, TGARCH or VS-GARCH models.² As GARCH(1,1) appears to fit financial data well in practice, we restrict our attention to the ANST-GARCH(1,1) specification but generalizations of equation (2) can be expressed straightforward. Furthermore, if we impose the restrictions $\omega_2 = 0$ and $\beta_2 = 0$, the model becomes that independently proposed by Hagerud (1997) and González-Rivera (1998). Note that many others GARCH formulations can be obtained from the equation (2) as a limiting case simply imposing some restrictions to the coefficients and considering the properties of the transition function.

The term ψX_t is not present in the parametrization of Anderson *et al.* (1999), but it is considered here because we investigate the properties of the model when exogenous variables are added to the conditional variance equation, like Gallo and Pacini (1998) consider in the context of GARCH and EGARCH models. In particular, we include in the original model two technical indicators known as *Infra-Daily Volatility (IDV)* and *Overnight Indicator (ONI)* and defined as

$$IDV_t \equiv \frac{P_t^H - P_t^L}{P_t^C}, \quad ONI_t \equiv \left(\frac{P_t^O - P_{t-1}^C}{P_{t-1}^C} \right)^2, \quad (4)$$

where P_t denotes respectively the highest (H), the lowest (L), the opening (O) and the closing (C) price on day t . The other two variables that we consider are a daily implied volatility index, called *VXN* and created by the CBOE in 1995, which is a weighted index of American implied volatilities calculated from eight near-the-money NASDAQ-100 call and put options with a time-to-expiration close to 30 days,³ and the daily *Volume* as a proxy for new information arrival in the market. A lagged value for the regressors *Volume*, *IDV* and *VXN* is included to avoid problems of simultaneity among the variables.

2. Empirical results

The aim of our empirical study is to investigate the properties of the ANST-GARCH model when exogenous variables are added to the conditional variance equation. To do this, we compare different specifications considering both the *News Impact Curve* (Engle and Ng (1993)), and the persistence reduction of the estimated volatility. Table 1 reports empirical results for the daily returns of Yahoo Inc stock and for the sample period from April 14, 1996 to March 31, 2000 (1034 observations); the mean of the return process

²See Anderson *et al.* (1999) for details.

³See the CBOE web site for more information: <http://www.cboe.com/MktData/vix.asp>.

Table 1: Estimation of GARCH(1,1) model and various ANST-GARCH(1,1) models for the Yahoo Inc stock. Robust standard errors are reported in parenthesis.

	GARCH	ANST-GARCH	$Volume_{t-1}$	IDV_{t-1}	ONI_t	VXN_{t-1}
μ	0.0044 (0.0016)	0.0039 (0.0015)	0.0038 (0.0015)	0.0036 (0.0016)	0.0017 (0.0015)	0.0037 (0.0015)
ω_1	0.0003 (8.79×10^{-5})	0.0003 (0.0002)	0.0004 (0.0002)	0.0003 (0.0003)	0.0005 (0.0002)	0.0001 (0.0002)
α_1	0.1264 (0.0217)	0.1994 (0.0379)	0.2110 (0.0414)	0.1604 (0.0648)	0.3650 (0.0917)	0.2088 (0.0416)
β_1	0.7561 (0.0466)	0.6466 (0.1141)	0.5886 (0.1235)	0.3769 (0.1710)	0.4007 (0.1292)	0.6032 (0.1246)
ω_2	-	-7.5×10^{-5} (0.0005)	-0.0002 (0.0005)	-0.0001 (0.0006)	0.0014 (0.0004)	-4.3×10^{-5} (0.0005)
α_2	-	-0.1427 (0.0456)	-0.1566 (0.0506)	-0.1856 (0.0717)	-0.3842 (0.1019)	-0.1525 (0.0504)
β_2	-	0.2381 (0.2308)	0.2933 (0.2384)	0.3134 (0.2910)	-0.4373 (0.1562)	0.2191 (0.2272)
γ	-	4.64 (7.82)	4.02 (6.82)	3.6 (6.67)	4.56 (5.27)	4.26 (7.92)
ψ	-	-	0.0086 (0.0066)	0.0124 (0.0034)	1.3736 (0.2326)	0.0080 (0.0042)
λ	0.8825	0.8937	0.8679	0.6012	0.3549	0.8453

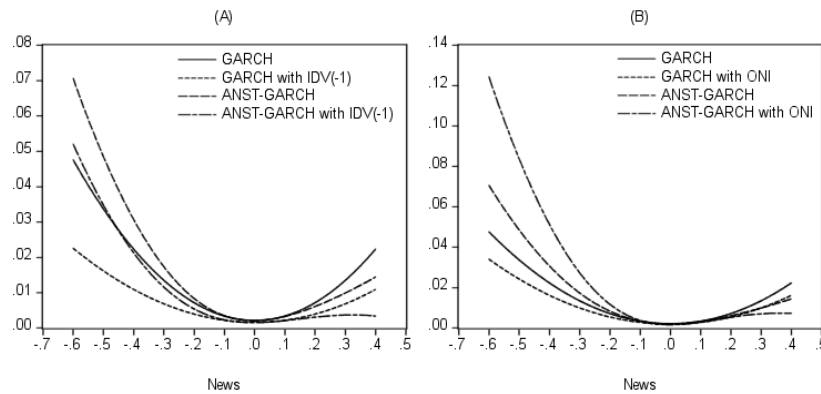
Table 2: Estimated persistence for the NASDAQ-100 Index.

	GARCH	ANST-GARCH	IDV_{t-1}	ONI_t
λ	0.9899	0.4969	0.4391	0.4881

has been specified by $r_t = \mu + \epsilon_t$. The parameter λ measures the persistence estimated in each models: $\alpha_1 + \beta_1$ in the standard GARCH model and $\alpha_1 + \beta_1 + \frac{1}{2}(\alpha_2 + \beta_2)$ in an intermediate state of the ANST-GARCH (see Anderson *et al.* (1999)). The ANST-GARCH specifications have been estimated by the LogL Object in EVIEWS 3.1 using the Marquardt algorithm; these routines are also available from the author upon request. Note that it is notoriously difficult to obtain a precise estimate of the smoothness parameter γ and often appears to be insignificant when judged by its t -statistic. This is due to the structure of the model, in particular to the partial derivative of σ_t^2 with respect to γ which arise to a large variance for the smoothness parameter (González-Rivera (1998)). Figure 1 shows the estimated *News Impact Curve* for the GARCH and ANST-GARCH models, both with and without the IDV_{t-1} and ONI_t variables.⁴ An analogous experiment has been repeated using the NASDAQ-100 Index for a longer sample period. The results, available only for IDV_{t-1} and ONI_t , confirm a big reduction in the estimated persistence (Table 2) and the *News Impact Curve* shows a pattern which is similar to the one of the Yahoo Inc stock. In particular, we find that the simple ANST-GARCH model shows a big reduction in the estimated persistence with respect to the GARCH specification; this might be interpreted as a suitable characterization of the regime-switching behavior.

⁴The estimated GARCH models with IDV_{t-1} and ONI_t are not reported here for reasons of space.

Figure 1: News Impact Curves for the estimated GARCH and ANST-GARCH, with and without IDV_{t-1} and ONI_t .



3. Concluding remarks

In this paper we have examined the properties of the ANST-GARCH class of models when exogenous regressors are added to the conditional variance equation. In particular, we have found that the parameter ψ is highly significant when the variables IDV_{t-1} and ONI_t are insert into the model;⁵ this is reflected in a modification of the shape of the *News Impact Curve* and in a big reduction of the estimated persistence. However, the impact of the lagged *Volume* is very low, similarly to other empirical research in the context of the traditional GARCH model; the same conclusions can be obtained if the VXN_{t-1} is considered. For these last two variables, no modifications of the *News Impact Curve* are displayed and also the persistence reduction is not significative. Further research are necessary to evaluate the effective presence of ANST-GARCH effects.

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⁵Only for the NASDAQ-100 Index, ONI_t is not significant.