

On the reliability of repairable systems: methods and applications

Affidabilità di sistemi riparabili: metodi ed applicazioni

Fabrizio Ruggeri, Antonio Pievatolo
CNR-IMATI, Via A.M.Ampère 56, I-20131 Milano
fabrizio@iami.mi.cnr.it, marco@iami.mi.cnr.it

Riassunto: I sistemi riparabili soggetti ad una riparazione minima sono caratterizzati dal fatto che la loro affidabilità rimane invariata dopo un guasto e la loro riparazione immediata. I guasti in tali sistemi sono spesso descritti mediante processi di Poisson non omogenei. Noi illustriamo l'uso di tali processi, prevalentemente nel contesto di due applicazioni industriali. La prima applicazione riguarda le fughe di gas in una rete urbana di distribuzione mentre la seconda considera i guasti di alcuni componenti nelle carrozze della metropolitana.

Keywords: Nonhomogeneous Poisson processes, reliability.

1. Introduction

The paper stems from our recent research and consulting activities on modelling failures in repairable systems, namely gas distribution networks and underground trains. The activity is based upon a cooperation among CNR-IMATI (formerly CNR-IAMI), the Department of Management Economics and Industrial Engineering (formerly part of the Department of Mechanical Engineering) at Politecnico di Milano and two of the major Italian gas and transportation companies. Historical data were provided by the companies and researchers from both CNR and Politecnico worked on them, with the remarkable help by students from Mechanical Engineering, Mathematics (Università degli Studi di Milano) and Scuola per le Applicazioni della Matematica nell'Industria (INdAM - Milano Bicocca).

The paper is organised as follows: an illustration of repairable systems, the most common models for them and the quantities of interest (Sections 2, 3 and 4, respectively) will be followed by a summary on our work on gas escapes and underground trains failures (Section 5 and 6, respectively). Finally, Section 7 will discuss shortly directions of our future research in the area.

2. Repairable systems

Systems subject to failures can be categorised into two groups: the nonrepairable and the repairable ones. When a car water pump fails, then the system *water pump* is nonrepairable since the pump must be replaced; looking at the system *car*, it is evident that the car can still run once the pump has been replaced: it is an example of repairable system. Repairable systems can be subdivided further by considering the repair strategy. Repairs could be *minimal* or more considerable. In the former case system reliability (i.e. the propensity to failures) after a failure is brought back to its status right before the failure,

whereas in the latter case reliability could be significantly reduced or increased. Reliability in the former case is usually told to be *bad as old* whereas *good as new* is used for the latter case. Some authors (e.g. Ascher and Feingold (1984)) prefer to denote the two cases as *same as old* and *same as new*, respectively, since old systems are not necessarily bad ones (and new ones could be bad, indeed). As a further category, we can consider if the repair is performed immediately after the failure or there is some elapsed time. In the latter case, availability of the system becomes important as well as its reliability.

3. Models for repairable systems

Nonrepairable systems are often described by the distribution of the time to first failure or their survival function. In repairable systems, the interest mainly relies on their behaviour over time, e.g. the expected number of failures or the probability of no failures in given intervals. Point processes (see, e.g. Thompson (1988)) are the natural tools used to describe failures in these systems. Under an instantaneous repair strategy, two different classes are considered: renewal processes (*same as new*) and nonhomogeneous Poisson processes (NHPP, *same as old*). The former processes consider the time between failures as i.i.d. random variables so that the system, subject to consistent replacements after each failure, is brought back to its initial reliability. Conversely, NHPPs keep their reliability unchanged after failures. The homogeneous Poisson process (HPP) is the only process which is both a NHPP and a renewal process.

In the paper we consider only NHPPs. Their use in modelling the reliability of repairable systems is illustrated in Rigdon and Basu (2000) and the references therein. We denote a NHPP by $N_t, t \geq 0$ or $N(t, s), 0 \leq t < s$, i.e. by the number of failures either in $(0, t]$ or in $(t, s]$, respectively. Its intensity function is given by

$$\lambda(t) = \lim_{\Delta t \rightarrow 0} \frac{P(N(t, t + \Delta t] \geq 1)}{\Delta t}.$$

The mean value function (m.v.f.) of the NHPP is given by $M(t) = \mathcal{E}N_t$. Note that, assuming no simultaneous failures, $\lambda(t)$ and $M(t)$ are related by $\lambda(t) = M'(t)$ and, consequently, by $M(t) = \int_0^t \lambda(u) du$.

We suppose that the system is observed in the interval $(0, y]$ and failures are recorded at $0 < t_1 < \dots < t_n \leq y$. y could be either a predetermined endpoint or the time of the $n - th$ (predetermined) failure. In the former case we talk about a *time truncated* experiment, whereas the latter is a *failure truncated* one. Under both experiments and the intensity function $\lambda(t) = \lambda(t; \theta)$, the likelihood function is

$$L(\theta; \underline{t}) = \prod_i^n \lambda(t_i; \theta) \exp\left\{-\int_0^y \lambda(t; \theta) dt\right\}. \quad (1)$$

4. Quantities of interest

Given the intensity function $\lambda(t; \theta)$, we are interested in the *usual* statistical analysis: point and interval estimation and hypothesis testing, from a frequentist and, mainly, from a Bayesian viewpoint. As an example, consider the *power law process* (PLP) which has been widely used for repairable systems. Its intensity function is $\lambda(t) = M\beta t^{\beta-1}$, with

$M, \beta, t > 0$. Estimation of β is very relevant since its value determines if (and how) reliability decays or grows over time ($\beta > 1$ and $\beta < 1$, respectively).

As typical quantities of interest in reliability, we consider the *reliability* $R(t, s) = P(\text{no failures in } (t, s])$, the *expected number of failures* $\mathcal{E}N(t, s)$ and the *intensity function* $\lambda(t)$ at t . The first two quantities are relevant in forecasting system reliability; typically t is supposed to be either 0 (when interested in a similar system from its start) or y (when interested in the behaviour of the system under consideration right after the end of the observed period). The third quantity is useful in problems like software prototype testing. Software is tested to reduce bugs in it but it must be released at some time, before it becomes obsolete. Its reliability during its commercial life is the same it had when released: it is therefore important to study $\lambda(t)$ at the release time.

We illustrate modelling and estimation issues in the next two sections, concerning two industrial applications. In particular, we present and compare parametric and nonparametric models, along with more complex models based upon superposition of NHPPs and upon failures recorded on a double scale (time and space).

5. Gas escapes

The gas distribution network considered in our case study was developed in a large urban area during the last century and thereby it is characterised by very different technical and environmental features (material, diameter of pipes, laying location, etc.). It consists of several thousands of kilometers of pipelines in low, medium and high pressure. The main concerns about the maintenance and replacement plans were related to the low-pressure network (20 mbar over the atmospheric pressure). As regards to the low-pressure pipelines, several materials have been used to develop the network: traditional cast iron, treated cast iron, spheroidal graphite cast iron, steel and polyethylene.

5.1 Cast iron pipelines

Cast iron pipelines have a higher failure rate than other materials, even ten times greater and cover more than a fourth of the whole network; they have been thoroughly studied in Cagno *et al.* (1998), Cagno *et al.* (2000a) and Cagno *et al.* (2000b). The structural failure of cast iron pipes is a rare phenomenon and their failure rate seems to be scarcely sensitive to wear, but mostly influenced by accidental stress. Due to these considerations, HPPs were used to model failures under 8 different technical and environmental conditions. Following a robust Bayesian approach, we combined data and experts' opinions and suggested a replacement strategy, quite robust with respect to reasonable changes in the priors. Priority in the replacements should be given to pipelines under traffic, specially those with small diameter and not deeply laid.

The failure data of the cast iron pipes have also been analysed in Fa-Si-Oen and Pievatolo (2000) by means of a generalised regression model with proportional intensity function, where the baseline intensity function was that of a PLP. A formal (frequentist) hypothesis testing on its shape parameter showed that an ordinary generalised linear model for Poisson counts is sufficient.

5.2 Steel pipelines

Steel pipelines have very strong mechanical properties (failures are very rare), but they

are easy prey for corrosive agents unless they are correctly protected. We consider 33 failures, due to corrosion, in the period 1978–1997 over a network of 275 kilometres. The cumulative number of failures is depicted in Fig. 2. More details on the data can be found in Fumagalli (1999).

Available data and experts' opinions led to identify three principal elements that may be related to the failures: the age of the pipe, the type of corrosion that leads to the rupture of pipe, and the lay location of the pipe. Here we consider only the first element, whereas the others have been analysed in Cagno *et al.* (1999).

The choice of a NHPP is justified since corrosion develops progressively, reducing the thickness of the pipe walls until the pipe breaks and the gas escapes.

5.2.1 Parametric models

Cavallo (1999) and Cavallo and Ruggeri (2001) considered a unique, parametric NHPP to model all the failures: they entertained the HPP, the PLP and, mainly, the logarithmic NHPP with $\lambda_\theta(t) = a \ln(1 + bt) + c$, $\theta := (a, b, c) \in \mathbf{R}_+^3$.

The m.v.f. of the logarithmic NHPP is given by

$$\Lambda_\theta(t) = \int_0^t \lambda_\theta(s) ds = \frac{a}{b}(1 + bt) \ln(1 + bt) + (c - a)t.$$

Given N_T failures, $\underline{y} = (y_1, \dots, y_{N_T})$, in $[0, T]$, the likelihood (1) becomes

$$L(\theta \mid \underline{y}, N_T) = (1 + bT)^{-a(1+bT)/b} e^{(a-c)T} \prod_{j=1}^{N_T} (a \ln(1 + by_j) + c).$$

MLE and Bayesian (under independent Gamma priors) estimates are compared in Table 1. Results on HPD interval and predictive distribution of N_t can be found in Cavallo (1999).

Table 1: *MLE and Bayesian estimates*

Parameter	MLE	Mean	Median	Mode	Variance
a	0.000	1.243	1.141	0.962	0.493
b	0.724	1.662	1.374	0.799	1.454
c	4.755	2.797	2.780	2.862	1.021

The MLE leads actually to a HPP, whereas the Bayesian approach favours the non-homogeneous component, as shown by posterior mean and variance of a . Cavallo and Ruggeri (2001) shows that the estimated m.v.f. of the HPP is closer to the sample m.v.f. than the one from the NHPP. The better performance of the HPP is confirmed by looking at the Bayes factor: the HPP is slightly better than the logarithmic NHPP and both are strongly better than the PLP. The Kolmogorov-Smirnov (K-S) test has been unable to reject the null hypothesis on the data coming from the logarithmic NHPP with the estimated parameters. The K-S test has been performed by transforming the data y_j so that the generalised residuals $\tau_j := \hat{\Lambda}(y_j)$, $j = 1, \dots, N_T$, are (under the null hypothesis) approximately arrival times from a HPP with unit intensity (see Ogata (1988)). In this case,

the interarrival times $\overline{v}_j := \overline{\tau}_j - \overline{\tau}_{j-1}$ are i.i.d. exponentially distributed, with mean 1, and we can apply the K-S test to $\overline{u}_j := 1 - \exp(-\overline{v}_j)$, which are uniformly distributed under the null hypothesis.

Therefore, we decide to relax the assumption of a logarithmic NHPP considering both a nonparametric model, to be “centred” at the best estimate of the NHPP (Section 5.2.2), and the superposition of NHPPs (Section 5.2.3).

5.2.2 Nonparametric models

Stemming from the paper by Lo (1982), Cavallo (1999) and Cavallo and Ruggeri (2001) considered the m.v.f. Λ as the distribution function of a random measure M , the intensity measure of the process.

Given a measure μ and a measurable subset B , we define $\mu B := \mu(B)$.

Definition 1 Let α be a finite, σ -additive measure on $(\mathbf{S}, \mathcal{S})$. The random measure μ follows a **Standard Gamma** distribution with shape α (denoted by $\mu \sim \mathcal{GG}(\alpha, 1)$) if, for any family $\{S_j, j = 1, \dots, k\}$ of disjoint, measurable subsets of $\mathbf{S}$, the random variables μS_j are independent and such that $\mu S_j \sim \mathcal{G}(\alpha S_j, 1)$, for $j = 1, \dots, k$.

Definition 2 Let β be an α -integrable function and $\mu \sim \mathcal{GG}(\alpha, 1)$. The random measure $\nu = \beta\mu$ follows a **Generalised Gamma** distribution, with shape α and scale β (denoted by $\nu \sim \mathcal{GG}(\alpha, \beta)$).

The Generalised Gamma distributions are conjugate for the Poisson processes.

Theorem 1 (Lo (1982)) Let $\underline{\xi} = (\xi_1, \dots, \xi_n)$ be n Poisson processes with intensity measure M . If $M \sim \mathcal{GG}(\alpha, \beta)$ a priori, then $M \sim \mathcal{GG}(\alpha + \sum_{i=1}^n \xi_i, \beta/(1 + n\beta))$ a posteriori.

In the next we consider the observations $\{y_{ij}, i = 1 \dots k_j\}_{j=1}^n$ from n Poisson processes $\underline{\xi} = (\xi_1, \dots, \xi_n)$ on $\mathbf{S}$. Note that y_{ij} denotes the failure time when k_j failures are recorded in the j -th system, $j = 1, \dots, n$, observed over $\mathbf{S} = [0, T]$.

Theorem 2 (Lo (1982)) Under the above assumptions and squared loss function, the Bayesian estimator of $M \sim \mathcal{GG}(\alpha, \beta)$ is given by the measure $\widetilde{M}$ such that

$$\widetilde{M}S = \int_{\mathbf{S}} \frac{\beta(x)}{1 + n\beta(x)} \alpha(dx) + \sum_{j=1}^n \sum_{i=1}^{k_j} \frac{\beta(y_{ij})}{1 + n\beta(y_{ij})} \mathbf{I}_S(y_{ij}), \quad \forall S \in \mathcal{S}.$$

Theorem 3 (Cavallo and Ruggeri (2001)) Given n Poisson processes $\underline{\xi}$ with common intensity function $M \sim \mathcal{GG}(\alpha, \beta)$, the Bayesian estimator of R , under squared loss function, is given by $\widetilde{R}$ such that, $\forall S \in \mathcal{S}$,

$$\widetilde{R}S = \exp \left\{ - \int_{\mathbf{S}} \ln(1 + \frac{\beta(x)}{1 + n\beta(x)}) \alpha(dx) - \sum_{j=1}^n \sum_{i=1}^{k_j} \ln(1 + \frac{\beta(y_{ij}) \mathbf{I}_S(y_{ij})}{1 + n\beta(y_{ij})}) \right\}.$$

We consider a nonparametric model “centred” at the logarithmic NHPP, whose m.v.f. is the Bayesian estimator $\hat{\Lambda}_\theta$, with intensity $\hat{\lambda}_\theta(s)$. Therefore, we choose $\alpha(ds) := \hat{\Lambda}_\theta(s)/\sigma ds$ and $\beta(s) := \sigma$, so that the measure EM has distribution function $\hat{\Lambda}_\theta$, and $MS \sim \mathcal{G}$ with mean $\hat{\Lambda}_\theta S$ and variance $\sigma \cdot \hat{\Lambda}_\theta S$, for any $S \in \mathcal{S}$.

Theorems 2 and 3 give the Bayesian estimators

$$\widetilde{MS} = \frac{\hat{\Lambda}_\theta S + \sigma \cdot \sum_{j=1}^n \xi_j S}{1 + n\sigma} \quad \text{and} \quad \widetilde{RS} = \left(1 + \frac{\sigma}{1 + n\sigma}\right)^{-\left(\hat{\Lambda}_\theta S + \sigma \sum_{j=1}^n \xi_j S\right)/\sigma}.$$

$\widetilde{M}$ is a weighted mean of the (prior) measure $\hat{\Lambda}_\theta$ and the n observed processes ξ_j ; the weight of the former decreases when σ increases and vice versa.

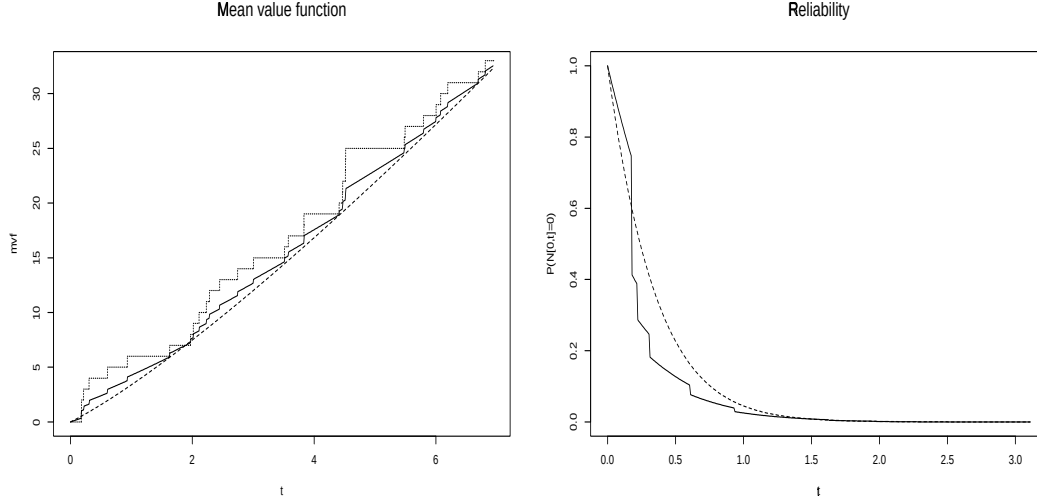


Figure 1: *Nonparametric (solid) and parametric (dashed) estimators and cumulative $N[0, t]$ (dotted)*

Figure 1 presents the nonparametric estimators of m.v.f. and reliability under a prior $\mathcal{GG}(\hat{\Lambda}_\theta/\sigma, \sigma)$, with $\sigma = 0.509$. It should be observed that the estimation of the m.v.f. improves significantly upon the parametric case. The finding is confirmed when computing the Bayes factor between the parametric and the nonparametric models, as shown in Cavallo and Ruggeri (2001).

5.2.3 A complex model

The previous models consider the pipelines as if they were installed at the same time. Since aging is relevant, we consider their installation date (actually, year). We split the network into subnetworks containing all those pipelines installed in the period (year) s ; by s we also denote the starting time of the corresponding period. Let $\mathcal{S}$ the set of all the periods in which pipelines were installed and let r be the number of its elements. More details are available in Pievatolo and Ruggeri (2000) and Pievatolo and Ruggeri (2001).

Suppose that both failure times t_i and installation dates δ_i of n failures in the interval $[T_0, T_1]$ are available; they are recorded in $\underline{t} = (t_1, \dots, t_n)$ and $\underline{\delta} = (\delta_1, \dots, \delta_n)$. For each $s \in \mathcal{S}$, consider the PLP N_s with intensity

$$\lambda_s(t; M_s, \beta_s) = l_s M_s \beta_s (t - s)^{\beta_s - 1} I_{[s, \infty)}(t),$$

where l_s is the length of the parts installed at s .

Combining the likelihoods for each PLP, it follows, from their independence, that the likelihood function for the system is given by

$$L(\underline{M}, \underline{\beta}; \underline{t}, \underline{\delta}) = \prod_{k=1}^n \beta_{\delta_k} l_{\delta_k} M_{\delta_k} (t_k - \delta_k)^{\beta_{\delta_k} - 1} e^{-\sum_{s=1}^r l_s M_s [(T_1 - s)^{\beta_s} - (s \vee T_0 - s)^{\beta_s}]}, \quad (2)$$

with $\underline{M} = (M_1, \dots, M_r)$ and $\underline{\beta} = (\beta_1, \dots, \beta_r)$.

For all s , we suppose $M_s \sim \mathcal{E}(\theta_M)$, $\beta_s \sim \mathcal{E}(\theta_\beta)$, with $\theta_M \sim \mathcal{E}(\tau_M)$ and $\theta_\beta \sim \mathcal{E}(\tau_\beta)$. Therefore, we consider $\underline{\beta}$ independent from $\underline{M}$, whereas the M_s 's and the β_s 's are exchangeable among themselves.

The posterior distribution is obtained applying an MCMC algorithm, based on a Gibbs sampler for all the parameters except for the β_s 's, updated sequentially by Metropolis steps. The sampled values from the posterior distribution are used to compute the reliability measures, as the the system reliability for the global network

$$R(T_1, u) = \exp\left\{-\sum_{s \in \mathcal{S}} M_s l_s \left[(u - s)^{\beta_s} - (T_1 - s)^{\beta_s}\right]\right\} I_{[s, \infty)}(u)$$

and the expected number of failures in the whole network in a future interval $(T_1, u]$

$$E\{N(T_1, u) \mid \underline{M}, \underline{\beta}\} = \sum_{s \in \mathcal{S}} M_s l_s \left[(u - s)^{\beta_s} - (T_1 - s)^{\beta_s}\right] I_{[s, \infty)}(u).$$

Setting $\tau_M = 0.2$ and $\tau_\beta = 0.7$, the 95% posterior probability intervals for the system reliability and for the expected number of failures in five years are estimated as $[9.64 \cdot 10^{-5}, 0.01]$ and as $[4.59, 9.25]$, respectively.

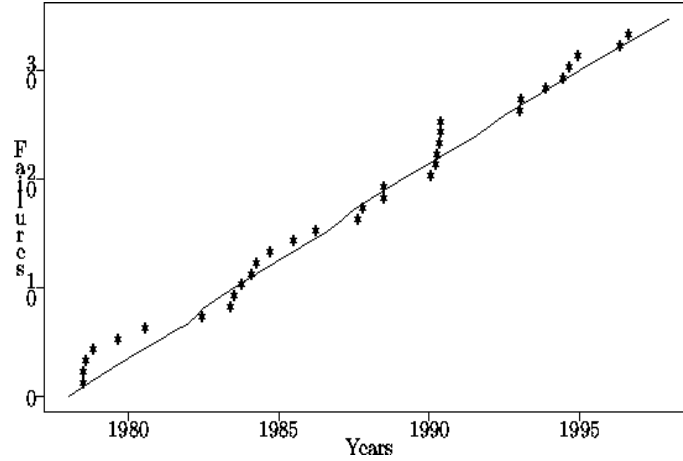


Figure 2: *Cumulative number of failures for the whole network compared to the estimated mean value function*

In Fig. 2 the (posterior) expected value of the mean value function

$$\sum_{s \in \mathcal{S}} M_s l_s \left[(t - s)^{\beta_s} - (T_0 - s)^{\beta_s}\right] I_{[s, \infty)}(u)$$

shows that there is a substantial agreement with the data, except for the two clusters of failures before 1980 and around 1990. We fear that there are some artifacts in the registration of the failures, with multiple recordings on the same day.

Finally, the installation dates of the failed pipelines are not always available, as well as most gas escapes before 1978. Missing and censored data are thoroughly analysed in Pievatolo and Ruggeri (2001).

6. Underground train failures

Argiento *et al.* (2002) considered the failures of the door opening system of 40 underground trains recorded between 6th April 1990 and 31st December 1998. When a failure took place, both the odometer reading and the date were recorded. In Fig. 3 we show the cumulative number of failures of all trains versus time; the asterisks indicate when each train has been put on service. A transient behaviour takes place during the first 500 days,

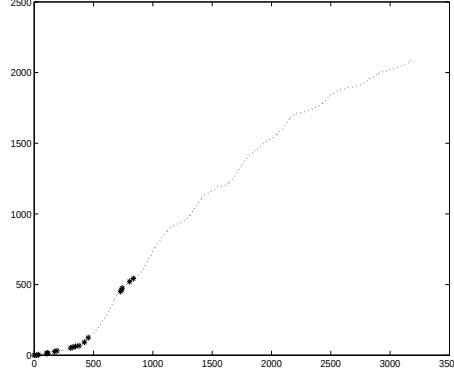


Figure 3: *Cumulative number of failures versus days for all trains*

when 35 trains are put on service; after this period, the number of trains is essentially constant and the cumulative number of failures gradually assumes a concave shape. Periodic oscillations can be noticed in the curve.

We consider both space and time via a marked Poisson process on the time scale, where the basic intensity function at time t depends on the expected number $g(t)$ of kilometers run at t , whereas k , the kilometers actually run at failure times, constitutes the mark.

Our model for a single train is based on the intensity function

$$\lambda(t; \theta_1, \theta_2) = \mu(g(t); \theta_1) s(t; \theta_2), \quad t \geq 0, \quad \theta_1 \in \Theta_1, \quad \theta_2 \in \Theta_2.$$

The conditional distribution of k , given a failure at t , is $\mathcal{N}(g(t), \sigma^2)$, and $s(\cdot; \theta_2)$ is the periodic component. After regressing the kilometers on time for many trains, we found that the functional form $E(k | t) = g(t) = at + bt^2$ is quite acceptable. The spectrum of the monthly number of failures of all trains suggests that only the harmonic corresponding to a period of one year is predominant. Therefore, we choose $s(t; \theta_2) = \exp\{\rho \cos(\omega t + \varphi)\}$, $\theta_2 = \rho \geq 0$, where only the amplitude ρ should be estimated, since $\omega = 2\pi/365$ (one-year frequency when time is measured in days), and the phase φ is known from the start date of each train.

Since the plot of the cumulative number of failures versus kilometers denotes early failures followed by a moderate reliability growth, we take

$$\mu(k; \theta_1) = \beta_0 \frac{\log(1 + \beta_1 k)}{(1 + \beta_1 k)}, \quad \theta_1 = (\beta_0, \beta_1), \quad \beta_0, \beta_1 \geq 0,$$

increasing up to its maximum point at $(e - 1)/\beta_1$ and then decreasing to zero.

MLE and confidence intervals for one train are reported in Table 2.

The fitting of the model can be assessed from Fig. 4, and can be considered satisfactory apart from that of $g(t)$, where systematic departures from the estimated curve can be seen

Table 2: MLE and 95% asymptotic confidence intervals for the (scaled) parameters

Parameter	MLE	C.I.	Parameter	MLE	C.I.
$a \times 10^{-2}$	1.209	[1.171, 1.247]	$b \times 10^2$	2.025	[1.862, 2.188]
$\sigma^2 \times 10^{-7}$	5.809	[4.214, 8.345]	$\rho \times 10$	3.234	[0.000, 6.779]
$\beta_0 \times 10^2$	7.358	[5.640, 9.076]	$\beta_1 \times 10^5$	2.239	[1.938, 2.540]

at a close examination. We plan to address the problem using a nonparametric model on the curve g .

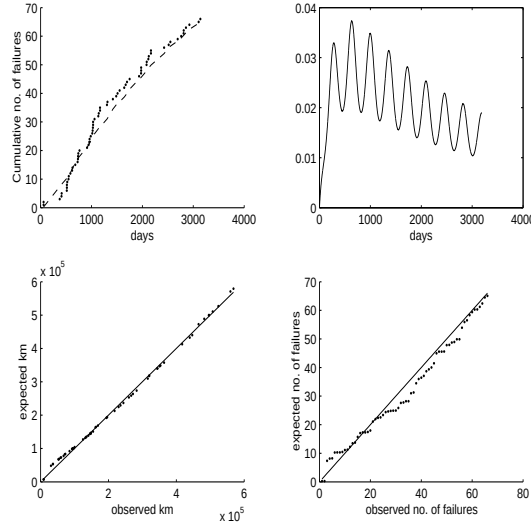


Figure 4: Diagnostic plots for one train: estimated mean value function and observed failures (top left), estimated intensity function (top right), expected odometer readings at failure times vs. observed readings (bottom left) and expected number of failures versus observed (bottom right)

7. Future research

We are currently working in directions aiming to improve the results found insofar. The nonparametric approach in Section 5.2.2 seems to be a very promising one: we are investigating the case of more general functions β . As regard the parametric approach in Section 5.2.1, we are considering classes of NHPPs defined as the solution of differential equations, as shown in Ruggeri (2000). We think the quality of the data on gas escapes will hardly lead to a very satisfactory fit with parametric models or their superposition as in Section 5.2.3. Finally, we are now working on a Bayesian nonparametric analysis of the train doors failures.

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