

Modelling techniques for extremes of stationary series: an application to rainfall data

Tecniche di analisi dei valori estremi di serie stazionarie: una applicazione a dati pluviometrici

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Riassunto: In questo lavoro vengono confrontati diversi metodi di analisi di valori estremi di serie stazionarie attraverso lo studio di osservazioni giornaliere di pioggia rilevate a Cittadella (Padova) nel periodo 1935-1999. Gli strumenti considerati vanno dal metodo classico dei massimi su periodi di tempo fissati a metodi più sofisticati basati sull'ipotesi di markovianità. All'aumento del livello di complessità del metodo corrisponde anche una maggiore capacità di includere nello studio osservazioni che hanno un contenuto informativo sul comportamento estremo della serie. Nell'ambito delle procedure markoviane, verrà sviluppato un modello, basato sulla coda di una distribuzione t bivariata, che rappresenta un'alternativa ai modelli proposti in letteratura. La sua semplice struttura e l'essere caratterizzato da parametri di facile interpretazione sono i principali vantaggi del nuovo modello, che tuttavia si dimostra essere flessibile e competitivo.

Keywords: Asymptotic independence; Bivariate t distribution; Block maxima; Extremal index; Extreme value theory; Markov chain; Threshold methods.

1. Introduction

Recent advances in extreme value theory have led to progressive improvements in modelling techniques for the extremal behaviour of time series. The early applications of extreme value theory were limited to the so-called block maxima approach (Gumbel, 1958) based on the limiting distribution of the maximum of independent and identically distributed (iid) random variables as a model for maxima over blocks of fixed length, typically one year.

The realization that this method is wasteful of information, when data relevant to extremes, other than the block maxima, are also available, has moved attention towards threshold methods (Davison and Smith, 1990). The idea is that all the observations above some high threshold u are extreme and should then be included in the analysis through a model provided by the asymptotic distribution of exceedances (Pickands, 1975). However, when exceedances of u are temporally close, account must be given to their serial dependence. In particular, it is common in environmental applications to observe series whose extreme values occur in clusters, generally as a consequence of meteorological persistence. The assumption of independence is obviously violated within a cluster, though it is often reasonable for extremes belonging to separate clusters.

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One possible solution is to identify clusters of exceedances of u and to keep only the maximum value for each cluster. This gives rise to the approach known as the peaks over threshold method which has two main drawbacks. First, it implies a wastage of data, since all exceedances, except the cluster maxima, are discarded. Second, it allows no understanding of the within-cluster behaviour, and hence of the stochastic properties of the observed phenomenon during an extremal episode. To overcome these limitations stronger assumptions on the temporal structure of the series are required.

A natural candidate for a class of processes that is sufficiently general to be of practical value, but sufficiently tractable that its extremal properties might be relatively easy to characterize, is the class of first-order Markov chains. Let $\{X_i\}_{i \geq 1}$ denote a stationary first-order Markov chain and $F(x_1, x_2)$ be the joint distribution of two consecutive variables. We say that $\{X_i\}_{i \geq 1}$ is asymptotically independent if X_1 and X_2 are asymptotically independent, that is

$$\lim_{x \rightarrow x^*} \Pr(X_2 > x | X_1 > x) = 0 \quad (1)$$

where x^* is the upper limit of the support of the common marginal distribution. When the limit in (1) is positive, the variables X_1 and X_2 , and the corresponding chain, are said to be asymptotically dependent. Extending results in Smith *et al.* (1997), Bortot and Tawn (1998) show that for asymptotically independent chains the degree of dependence between exceedances of x decreases as $x \rightarrow x^*$, with the extremal behaviour of $\{X_i\}_{i \geq 1}$ increasingly resembling that of an iid sequence. The class of asymptotically independent chains is nontrivial: besides important theoretical examples, such as Gaussian autoregressive processes (Sibuya, 1960), asymptotic independence is often encountered in environmental applications. To allow for the convergence to independence, as well as for the case of asymptotic dependence, Bortot and Tawn (1998) suggest modelling $F(x_1, x_2)$ for x_1 and x_2 both above a fixed high threshold u through a family introduced by Ledford and Tawn (1997, 1998). This class is derived by specifying a parametric form for a second-order characterization of the tail behaviour of bivariate distributions. However, some limitations can be identified in this procedure too: in representing an infinite-dimensional characterization with a specific parametric family, some flexibility is lost; in addition, the model itself is quite complex with parameters of difficult interpretability.

In this work we will use the analysis of a series of daily measurements of rainfall in Cittadella (Padova) over the period 1935–1999 to review and compare a range of modelling techniques for extremes of stationary series, from the earliest block maxima method to the more recent method based on the assumption of Markovianity. For the latter approach we will also explore the possibility of replacing Ledford and Tawn’s model for the joint upper tail of F with a simpler family having easily interpretable parameters, namely the tail of a bivariate t distribution. As shown by Embrechts *et al.* (2001), both asymptotic dependence and asymptotic independence are allowed within the bivariate t distribution with different choices of the degrees of freedom. We will show that, despite its simpler structure, for statistical applications the loss of flexibility with respect to Ledford and Tawn’s model is negligible.

The paper is structured as follows. In Sections 2 and 3 the approaches to the analysis of extremes of stationary time series outlined above are presented in greater detail. In Section 4 the model for extremes of Markov chains based on the tail of a bivariate t distribution is developed. Section 5 gives a description of the Cittadella rainfall data and compares results from different modelling techniques. Finally, Section 6 contains some concluding remarks.

2. Block maxima and threshold methods

The cornerstone of extreme value theory is the characterization of the limiting distribution of the sample maximum. If $\{Z_i\}_{i \geq 1}$ is a sequence of iid random variables with distribution function F_1 and there exist sequences $a_n > 0$ and b_n such that

$$\lim_{n \rightarrow \infty} \Pr \left\{ \frac{\max_{i=1, \dots, n} Z_i - b_n}{a_n} \leq x \right\} = \lim_{n \rightarrow \infty} F_1^n(a_n x + b_n) = G_1(x), \quad (2)$$

for a non-degenerate distribution function G_1 , then G_1 is a member of the so called generalized extreme value family

$$G_1(x) = \exp \left\{ - \left(1 + \xi \frac{x - \mu}{\tau} \right)_+^{-1/\xi} \right\}, \quad x \in \mathbb{R} \quad (3)$$

where μ is a location parameter, $\tau > 0$ a scale parameter, ξ a shape parameter and $a_+ = \max(a, 0)$. Used as an approximation, equation (3) provides a parametric model for the distribution of the maximum of a sufficiently large number of iid observations.

The above result can be extended to stationary series. If $\{X_i\}_{i \geq 1}$ is a stationary sequence having marginal distribution F_1 and $\{Z_i\}_{i \geq 1}$ is an iid sequence with the same marginal distribution F_1 for which the limit (2) holds then, subject to a restriction on long-range dependence (Leadbetter *et al.*, 1983),

$$\lim_{n \rightarrow \infty} \Pr \left\{ \frac{\max_{i=1, \dots, n} X_i - b_n}{a_n} \leq x \right\} = \lim_{n \rightarrow \infty} F_1^{n^\theta}(a_n x + b_n) = G_1^\theta(x), \quad (4)$$

Thus, convergence is guaranteed, but to G_1^θ in place of G_1 . The extremal index θ , $0 \leq \theta \leq 1$, has an interpretation in terms of high-level clustering of the X_i sequence. Specifically, θ^{-1} is the limiting mean number of threshold exceedances in clusters of extreme values (Hsing *et al.*, 1988).

Equations (3) and (4) constitute the basis of the block maxima approach. Note that G_1^θ still belongs to the generalized extreme value family (3), though the location and scale parameters will generally be different from those of G_1 . This implies that, even in the presence of temporal dependence, equation (3) can be used to approximate the distribution of maxima over blocks of observations of fixed, but sufficiently long, length. Once the generalized extreme value distribution is estimated from block maxima, it can be interrogated to infer the extremal properties of the process and to predict events that are more extreme than those observed. One of the quantities of main interest in applications is the T year return level. Loosely, this is the level x_T which is expected to be exceeded on average once every T years. More precisely, x_T is the $1 - 1/T$ quantile of the annual maximum distribution. If blocks are chosen to correspond to a time period of length one year, an estimate of x_T is the $1 - 1/T$ quantile of the fitted generalized extreme value distribution.

The basic model for threshold methods is the generalized Pareto family. Under the same conditions that guarantee convergence in (2), it can be shown (Pickands, 1975) that F_1 can be approximated above a high threshold u by

$$H(x) = 1 - \lambda \left(1 + \xi \frac{x - u}{\sigma} \right)_+^{-1/\xi}, \quad x > u \quad (5)$$

where the shape parameter ξ is the same as in (3), $\sigma > 0$ is a scale parameter and $0 < \lambda < 1$. The class of distributions defined by (5) is called the generalized Pareto family.

The most immediate application of the generalized Pareto distribution is to fit the parametric model (5) to all the observations exceeding a fixed high threshold u . However, if exceedances occur in clusters, approximation (5) is still valid marginally, but the general theory provides no specification for the joint distribution of neighboring exceedances. The solution that gives rise to the peaks over threshold method is to assume independence between clusters and estimate H only from cluster maxima. From (5) and the second equality of equation (4) we have that the distribution of the annual maximum M_y can be approximated by

$$\Pr(M_y \leq x) \approx H^{m\theta}(x), \quad x > u \quad (6)$$

where m is the number of time units in one year. It follows that estimates for return values can be derived from the peaks over threshold method by solving the equation

$$\hat{H}^{m\hat{\theta}}(x_T) = 1 - 1/T \quad (7)$$

where $\hat{H}$ is the estimated generalized Pareto distribution and $\hat{\theta}$ is the empirical estimate of the extremal index obtained as the reciprocal of the mean number of exceedances of u within a cluster.

3. Markov models

In the peaks over threshold method dependence between exceedances is effectively removed, but at the price of wasting information which is relevant to extremes and preventing inference on the within-cluster behaviour. Threshold methods based on the assumption of Markovianity aim to include in the analysis all the observed exceedances by providing their joint distribution. The drawback is that stronger assumptions are made on the underlying process.

Denote by $\{X_i\}_{i \geq 1}$ a stationary first-order Markov chain with continuous state space and by $F(x_1, x_2)$ the joint distribution of two consecutive variables. Smith (1992) shows that, under regularity conditions, the extremal properties of $\{X_i\}_{i \geq 1}$ are completely determined by the limiting behaviour of $F(x_1, x_2)$ for x_1 and x_2 both large. This justifies studying the extremes of the chain by focusing on the joint upper tail of F .

Ledford and Tawn (1997, 1998) show that for x_1 and x_2 above some high threshold u the following approximation holds

$$F(x_1, x_2) \approx H(x_1) + H(x_2) - 1 + \mathcal{L}\{t(x_1), t(x_2)\}t(x_1)^{-c_1}t(x_2)^{-c_2}. \quad (8)$$

where $t(x) = -1/\log H(x)$, $\mathcal{L}$ is a bivariate slowly varying function (Bingham *et al.*, 1987) and $c_1, c_2 > 0$, with $c_1 + c_2 \geq 1$. Note that the right-hand side of (8) has common generalized Pareto (5) margins.

Let $\bar{\chi} = 2(c_1 + c_2)^{-1} - 1$, $\bar{\chi} \in (-1, 1]$. From (8) we derive that for asymptotically dependent chains $\bar{\chi} = 1$. For $\bar{\chi} < 1$ we have asymptotic independence, with smaller values of $\bar{\chi}$ leading to faster convergence to 0 in (1). Thus, within asymptotic independence, $\bar{\chi}$ provides a measure of dependence at extreme levels which increases with dependence strength (Coles *et al.*, 1999). As an example, for a Gaussian autoregressive process with lag 1 autocorrelation $|\rho| < 1$ it can be verified that $\bar{\chi} = \rho$ (Ledford and Tawn, 1997).

The difficulty in working with approximation (8) lies in its generality: the class of functions $\mathcal{L}$ admits no finite parametrization. Consequently, for inference, some restrictions must be made. Ledford and Tawn (1997) propose the following parametric specification

$$\mathcal{L}(r_1, r_2) = a_0 + \frac{r_1 + r_2}{(r_1 r_2)^{1/2}} \left\{ 1 - \frac{r_1 r_2}{r_1 + r_2} (r_1^{-1/\alpha} + r_2^{-1/\alpha})^\alpha \right\}, \quad r_1, r_2 > 0, \quad (9)$$

with $a_0 \geq 1$ and $0 \leq \alpha \leq 1$. With this simplification, the joint tail model (8) can be fitted to observed transitions above the threshold u . Since no model is specified for $F(x_1, x_2)$ when one or both variables are below u , model parameters are estimated through maximum likelihood by including censored contributions for transitions which are either partially or completely below the bivariate threshold (u, u) (Smith *et al.*, 1997; Bortot and Tawn, 1998).

Once the model is estimated, various extremal aspects, such as the extremal index θ or the distribution of functionals of exceedances, can be inferred. This is achieved by simulating clusters of extreme events. When the fitted chain is asymptotically independent, this procedure allows us to describe the convergence of the tail of the chain to its limiting form, i.e. the tail of an iid sequence, and to derive summaries of the extremal behaviour that vary with the level. To clarify this point, it is easy to verify that for an asymptotically independent chain, the extremal index θ is equal to 1, so that in the limit there is no clustering of extreme values. However, at a finite level x exceedances of x form clusters whose size decreases to 1 as x increases. Hence, a threshold-dependent equivalent of the extremal index $\theta(x)$ gives a more accurate representation of extremal dependence at finite levels than its limiting value $\theta = 1$. Similar arguments hold for other summary statistics of the tail behaviour.

4. The bivariate t tail model

Formulation (9) covers a range of asymptotically dependent and asymptotically independent chains as special cases, but it also leads to some loss of flexibility with respect to the general characterization (8). For example, chains with bivariate Gaussian or bivariate t consecutive variables are excluded. Another disadvantage is the difficulty of parameter interpretability.

An alternative model that overcomes some of the limitations of Ledford and Tawn's specification is obtained from a marginal transformation of the tail of a bivariate t distribution. A bivariate random vector $\mathbf{T} = (T_1, T_2)'$ which admits the stochastic representation

$$\mathbf{T} =_d \frac{\sqrt{\nu}}{\sqrt{S}} \mathbf{Z}$$

where $S \sim \chi_\nu^2$ and $\mathbf{Z}$ is a standard bivariate Normal distribution with correlation coefficient ρ , S and $\mathbf{Z}$ independent, has standard bivariate t distribution with ν degrees of freedom and correlation coefficient ρ for $\nu > 2$. If $0 < \nu \leq 2$ the covariance matrix of $\mathbf{T}$ is not defined and ρ can be interpreted as a shape parameter. However, for simplicity, in both cases we will refer to ρ as the correlation parameter. The univariate margins of $\mathbf{T}$ are identically distributed according to a univariate t distribution with ν degrees of freedom. In the following we will work with the reparametrization $\phi = 1/\nu$.

Embrechts *et al.* (2001) show that $\mathbf{T}$ allows for both asymptotic dependence and asymptotic independence. In particular, they derive that

$$\lim_{x \rightarrow \infty} \Pr(T_2 > x | T_1 > x) = 2 \left\{ 1 - t_{\frac{\phi+1}{\phi}} \left(\sqrt{(1/\phi + 1)(1 - \rho)/(1 + \rho)} \right) \right\}$$

where $t_{1/\phi}$ denotes the distribution function of a univariate t distribution with $1/\phi$ degrees of freedom. From this, it is seen that dependence at extreme values is increasing in ρ and in ϕ . Asymptotic independence is obtained in the limit as $\phi \rightarrow 0$ for $\rho < 1$. Since this case corresponds to the bivariate Gaussian distribution, we know from the previous section that $\bar{\chi} = \rho$. From these arguments a model for the tail of F which covers both asymptotic dependence and asymptotic independence can be obtained in the following manner. Let $t_{1/\phi}^{(2)}(\cdot, \cdot; \rho)$ denote the distribution function of a bivariate t with $1/\phi$ degrees of freedom and correlation coefficient ρ . For x_1 and x_2 above some high threshold u we model $F(x_1, x_2)$ with

$$G(x_1, x_2) = t_{1/\phi}^{(2)}(\psi(x_1), \psi(x_2); \rho) \quad x_1, x_2 > u \quad (10)$$

where $\psi(x) = t_{1/\phi}^{-1}\{H(x)\}$. This is simply the tail of a bivariate t distribution transformed marginally to have generalized Pareto margins. For $\phi \rightarrow 0$ it reduces to the bivariate Gaussian tail model considered in Bortot *et al.* (2000).

The proposed model (10) has a simple structure and easily interpretable parameters. In addition, unlike model (8)-(9), it has an immediate multivariate extension that can be implemented for the study of higher-order Markov chains. One drawback is that for the bivariate t tail model variables X_1 and X_2 are exchangeable, implying time-reversibility of the chain. In the next section we will investigate the flexibility of model (10) through an application to the Cittadella data.

5. Application to Cittadella rainfall data

The data analyzed are measurements (mm) of daily rainfall recorded in Cittadella (Padova) from the 1st of January 1935 to the 31st of December 1999. The whole of 1987 is missing leaving 64 years' observations. This series is part of a larger set of rainfall data recorded at various stations on the catchment basin of Venice lagoon (Rama, 2000). Extremal episodes occurring at these stations can determine an increase of the sea level in Venice and hence affect the safety of the city. Consequently, interest is in quantifying the quantity of water flowing from the land to the sea, especially during severe meteorological events.

A preliminary analysis of the Cittadella series reveals a strong seasonal variation, with the most extreme events concentrating in the autumn period. Since other seasons give little contribution to severe episodes, we decided to focus attention on the autumn months, within which the process appears stationary. In addition, the study carried out by Rama (2000) would suggest the absence of a long-term trend, so we also assume stationarity over years.

One feature observed in the Cittadella data is the presence of clusters of extreme values. To illustrate this, in Figure 1 a section of the series is plotted together with a horizontal line representing the 0.94 quantile of the whole data set. The analysis of the

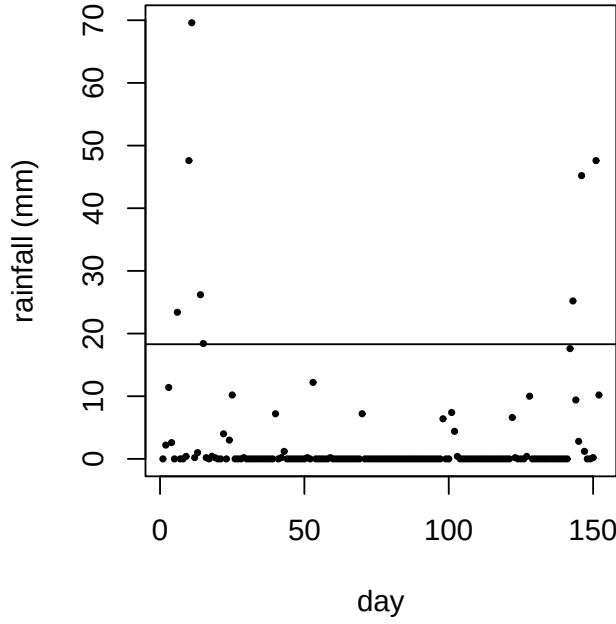


Figure 1: A section of the Cittadella rainfall series. The straight line represents the 0.94 marginal quantile.

extremal behaviour of the process must therefore take into account serial dependence between exceedances.

The first step was to apply the block maxima approach. We chose blocks of length one year and from the 64 annual maxima derived maximum likelihood estimates of the parameters of the generalized extreme value distribution (3). In particular, the estimate of the shape parameter (standard error in parentheses) is $\hat{\xi} = 0.043 (0.1)$. Thus, data support the simplification $\xi = 0$, which corresponds to the Gumbel distribution. The Gumbel distribution is also the basic model used by hydrologists for annual maximum rainfall.

Application of the peaks over threshold method requires identification of clusters. This step has components of arbitrariness that affect results. We considered different definitions of cluster to then find that clusters terminating after 3 consecutive observations have fallen below the 0.94 marginal quantile were giving more stable results. This gives rise to 333 events. The corresponding maximum likelihood estimate of the shape parameter of the generalized Pareto distribution (5) is $\hat{\xi} = 0.013 (0.056)$, indicating a slightly lighter tail than the annual maxima approach.

Finally, Markov procedures based on both Ledford and Tawn's model (8)-(9) and the bivariate t tail model (10) were applied. For both procedures, stability in dependence and marginal parameters was observable over the 0.94 marginal quantile, which was therefore selected as the threshold u .

For Ledford and Tawn's model, exchangeability, i.e. $c_1 = c_2$, was supported by a likelihood ratio test. For the reduced model, maximum likelihood estimates of the shape parameter of the marginal distribution and of the dependence parameter $\bar{\chi}$ are $\hat{\xi} = -0.011 (0.047)$ and $\hat{\chi} = 0.24 (0.041)$, respectively. The estimated value of $\bar{\chi}$ reveals asymptotic independence.

For the bivariate t tail model the estimate of ϕ is $\hat{\phi} = 0.00034$. Testing whether the data support the assumption of asymptotic independence, that is whether $\phi = 0$, against

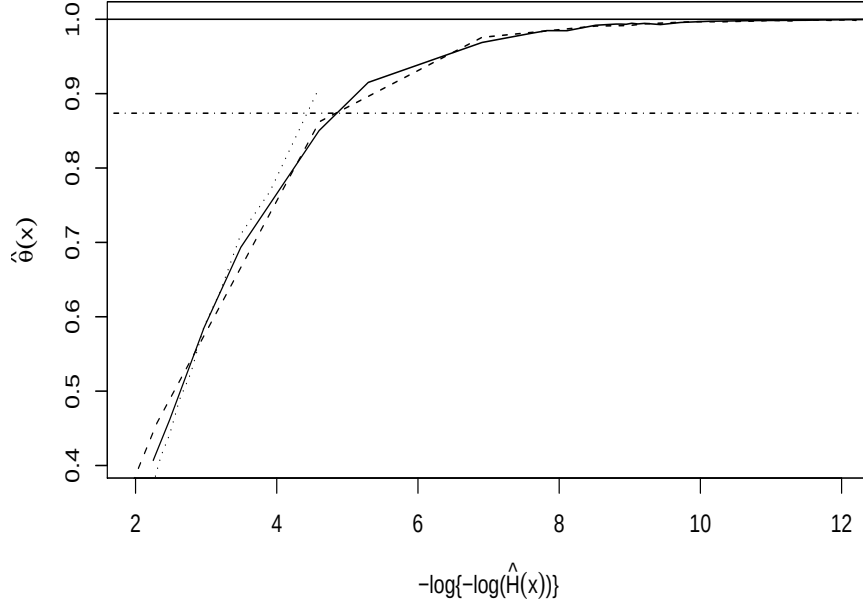


Figure 2: Estimates of $\theta(x)$ for large x . The solid line corresponds to Ledford and Tawn's model, the dashed line to the bivariate Gaussian tail model, the dotted line to the empirical estimates. The solid straight line represent the limiting value $\theta = 1$. The dot-dashed line is the estimated value of θ under asymptotic dependence.

the alternative hypothesis $\phi > 0$ is a nonregular problem since the parameter falls on the boundary of the parameter space under the null hypothesis. Standard asymptotic results of Self and Liang (1987) for the behaviour of the likelihood ratio test in boundary problems can be applied leading to the test statistic asymptotically being 0 with probability 1/2 and having a χ_1^2 distribution otherwise. The resulting p -value is 0.85, so that the reduction to a bivariate Gaussian tail model can be accepted. For the simplified model, estimates of the shape parameter of the generalized Pareto distribution and $\bar{\chi}$ are, respectively, $\hat{\xi} = -0.0068$ (0.046) and $\hat{\chi} = \hat{\rho} = 0.31$ (0.036), showing a good agreement with results obtained under Ledford and Tawn's model. To verify the suitability of the assumption of first-order Markovianity we fitted the trivariate analogue of the bivariate Gaussian tail model to consecutive triples of extremes. We found that the additional terms gave no significant contribution to the model.

To illustrate the decrease in dependence at extreme levels for the fitted first-order chains, in Figure 2 estimates of the threshold-dependent extremal index $\theta(x)$ for large x are plotted against $-\log\{-\log(\hat{H}(x))\}$ for Ledford and Tawn's model and for the bivariate Gaussian tail model. The limiting value θ is represented by a horizontal line at 1. Also shown are the empirical estimates of $\theta(x)$ obtained as the reciprocal of the sample mean cluster size at the level x using the same cluster definition as for the peaks over threshold method, i.e. terminating a cluster when 3 observations have fallen below x . There is a good agreement between the empirical and the model-based estimates. Figure 2 contains also the threshold-independent estimate of θ derived from Ledford and Tawn's model by setting $\bar{\chi} = 1$, that is by imposing asymptotic dependence. This value seems inconsistent with the empirical findings.

To synthesize comparisons between the different modelling procedures, Table 1 contains estimates of return values for 20, 50, 64, 100, 500 and 1000 years for the annual

Years	Annual maxima	Peaks over threshold	Ledford and Tawn	Bivariate Gaussian model	Ledford and Tawn ($\bar{\chi} = 1$)
20	89.5 (7.1)	86.6 (6.5)	89.9 (5.4)	90.1 (5.3)	94.1 (7.0)
50	103.7 (11.4)	100.6 (9.6)	102.8 (8.6)	103.2 (8.3)	109.4 (10.2)
64	107.7 (12.8)	104.3 (10.6)	106.2 (9.7)	106.7 (9.4)	113.6 (11.2)
100	114.9 (15.7)	111.2 (12.5)	112.4 (11.6)	113.0 (11.2)	121.2 (13.2)
500	142.0 (29.6)	136.3 (21.0)	134.4 (20.8)	135.6 (19.6)	149.4 (22.2)
1000	154.3 (37.4)	147.3 (25.4)	143.7 (23.9)	145.2 (23.5)	161.8 (26.9)

Table 1: Estimates of 20, 50, 64, 100, 500 and 1000 year return levels (mm) for the annual maxima approach, the peaks of threshold method and Markov procedures based on Ledford and Tawn’s model, the bivariate Gaussian tail model and Ledford and Tawn’s model under the restriction $\bar{\chi} = 1$. Standard errors are given in parentheses.

maxima approach, the peaks over threshold method and the Markov models. The 64 year return level was included because it is directly comparable with the maximum rainfall measurement recorded in Cittadella, i.e. 106.4 mm. For the asymptotically independent Markov models estimates of return values were derived from equation (7), but on replacing $\hat{\theta}$ with the estimated value of $\theta(x_T)$. The resulting equation was solved numerically using a smoothed curve interpolating estimates of $\theta(x)$. For all procedures, standard errors were obtained via the delta method. With the exception of the asymptotic dependent model, which seems to overestimate return values, there is overall a good agreement between all threshold methods, especially for the Markov procedures. Ledford and Tawn’s model and the bivariate Gaussian tail model give estimates of the 64 year return value that are very close to the observed one. They also produce the narrowest confidence intervals, because of the increased exploitation of within-cluster data. Comparison with the annual maxima approach highlights similarities up to the 100 year return values, though some discrepancies can be observed for longer periods of time. Note, however, that the analysis based on annual maxima yields very variable estimates.

6. Conclusions

The analysis carried out on the Cittadella data shows a general robustness of results to the specific modelling procedure adopted. However, estimate precision increases with the capacity of the model to include more information relevant to the extremal behaviour, as long as dependence between exceedances is correctly accounted for. So, for instance, Table 1 highlights the risks of using an asymptotic dependent model to describe an asymptotically independent chain.

Within the Markov framework, the study suggests that the proposed bivariate t tail model, despite its simple structure, is competitive with respect to Ledford and Tawn’s model (8)-(9). In addition, its straightforward extension to higher dimension has enabled us to verify the assumption of first-order Markovianity for the Cittadella series.

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