

A nonparametric estimation of the overflow probability with long-range dependent input¹

*Stima Non Parametrica Della Probabilità Di Overflow Con Ingressi A
Memoria Lunga.*

Pier Luigi Conti
Dipartimento di Statistica, Probabilità
e Statistiche Applicate
Università degli Studi di Roma "La Sapienza"

Livia De Giovanni
Dipartimento SEGeS -
Università del Molise

P.le A. Moro 5; 00185 Roma uniroma1.
pierluigi.conti@uniroma1.it

Via F. de Sanctis; 86100 Campobasso
degiovanni@unimol.it

Riassunto: in questo lavoro si considera un sistema a coda in tempo discreto $G/G/1$ (General arrivals/General service times/1 server), utile per la valutazione di prestazioni di sistemi di trasmissione asincroni, in particolare sistemi ATM (Asynchronous Transfer Mode). L'attenzione è rivolta alla probabilità di coda del "lavoro residuo" (unfinished work), in quanto equivalente alla probabilità di overflow. Si introduce una stima non parametrica, sia puntuale che intervallare, di un'approssimazione della probabilità di coda del lavoro residuo in equilibrio. Contrariamente alle ipotesi usuali, si assume una forma del tutto generale per la funzione di correlazione del processo di arrivi. Le stime ottenute sono applicate a dati reali di telecomunicazioni.

Keywords: Queues, Performance, Telecommunication, Long-Range Dependence.

1. Introduction

Asynchronous Transfer Mode (ATM) is a technology used to deploy broadband integrated service digital networks. Moreover standards and corresponding implementations already exist to run IP (Internet Protocol) over ATM, *i.e.* for transmitting IP datagrams in the form of ATM cells. Fixed length ATM cells are routed from sources to destinations through ATM nodes, which interconnect input links with output links through a switching fabric characterised by queues (buffers) on the output links to overcome the competition among cells requiring simultaneously the same output link. Each connection is characterised by a Quality of Service requirement.

In this paper a discrete-time $G/G/1$ (General arrivals/General service times/single server) queueing model is considered. The main contribution is a nonparametric estimate and

¹ Il lavoro è stato svolto con il contributo finanziario del Ministero dell'Istruzione, dell'Università e della Ricerca.

related asymptotic confidence intervals of an approximation of the unfinished work tail probability (Addie 1999). The tail probability of the unfinished work is equivalent to the buffer overflow probability, and hence closely related to the cell loss probability, one of the most important Quality of Service parameter in ATM/IP networks. A general service time distribution is commonly assumed in the literature. The situation is different for the arrival process. As shown by recent measurements in telecommunication networks, the increments of the arrival process are characterised by Long Range Dependence (LRD). As a consequence, a general autocorrelation function for the increments of the arrival process is allowed. The obtained estimate is evaluated on real data collected by Telecom Italia.

2. Model description

In this paper, we consider a single server queue with an infinite waiting room. No parametric assumptions are made on the distributions of inter-arrival and service time. The time is divided into consecutive intervals (time slots). Let V_n be the unfinished work of the queue at the beginning of n -th time interval, *i.e.* the amount of work remaining to be done at time n (Beneš, 1962). The unfinished work obeys the recurrence formula (Addie, 1999):

$$V_{n+1} = \max(0, V_n + Y_n), \quad n \geq 1$$

where $Y_n = A_n - B_n$ is the net input process, A_n, B_n being the number of cell arrivals in a time slot and the work that can be processed in a single time slot, respectively.

The random variate (r.v.) V_∞ represents the unfinished work in steady state. The load coefficient is $r = E[A_\infty] / E[B_\infty]$. The random variate V_∞ possesses a proper distribution if and only if $r < 1$. If $r \geq 1$, then $P(V_\infty > K)$ is equal to 1 for every $K > 0$. Apart from special cases, the exact distribution of V_∞ is not known. An approximation of the tail probabilities of V_∞ has been proposed (Addie 1999), based on the following assumptions:

Assumption (i). The input process $(Y_n; n \geq 1)$ is a stationary Gaussian process.

Assumption (ii). $\text{Var}(Y_1 + \dots + Y_n) \sim n^{2H}$, with $1/2 \leq H < 1$.

Assumption (i) is met when the input process $(Y_n; n \geq 1)$ is obtained by superimposing independent traffic processes satisfying the usual conditions of the functional central limit theorem. For instance, in telecommunications the (aggregated) input process is obtained by superposition of traffic streams produced by a large number of independent sources simultaneously active. As far as hypothesis (ii) is concerned, it simply means that the input process exhibits long range dependence. Statistical analysis of real traffic patterns shows that the process $(Y_n; n \geq 1)$ is actually characterized by long range dependence. Addie's formula, obtained *via* Wiener-Hopf factorization technique (Addie, 1994) reads as follows:

$$P(V_{\infty} > K) \approx A(m, s^2, H) \\ = \frac{\sqrt{2p}}{s} \Psi(-m, s) \exp \left\{ -\frac{1}{2s^2} (1-H)^{-2} \left(-\frac{m(1-H)}{H} \right)^{2H} K^{2(1-H)} \right\} I_{(m < 0)} + I_{(m > 0)} \quad (1)$$

In formula (1) m and s^2 are the mean and variance of Y_n , respectively, H is the Hurst parameter, $I_{(\cdot)}$ is the indicator function and K is the buffer size. The function $\Psi(\cdot, \cdot)$ is:

$$\Psi(x, y) = \left(\frac{y}{\sqrt{2p}} \right) \exp \left\{ -\frac{x^2}{2y^2} \right\} - \frac{x}{2} \left(1 - 2/\sqrt{p} \int_0^{x/(\sqrt{2y})} e^{-t^2} dt \right) \quad x \in \mathbb{R}, \quad y > 0$$

The problem we deal with in the present paper is the estimation of (1) on the basis of a sample of size n , $Y_1, \dots, Y_n$, say. We stress that in telecommunication applications the “service times” B_n are constant, and equal to the number of cells that can be transmitted in a time interval. Each time interval corresponds to a cell-time, so B_n ’s are equal to 1. Y_n is equal to $A_n - 1$, A_n denoting the amount of cells entering the system in the n -th time slot.

3. Asymptotic results and applications to real data

The basic idea of the present paper is to consider a plug-in estimate of the overflow probability, based on estimates of m , s^2 and H . As estimators of s^2 and H we take the Whittle’s approximate maximum likelihood estimators for Gaussian processes (Beran, 1994), $\hat{s}^2, \hat{H}$, say. To obtain semiparametric estimates, Whittle’s estimators of s^2 and H are obtained for the aggregated process $Y_j^p; j = 1, \dots, n/p$ and are based on the spectral density function of the Fractional Gaussian Noise (Beran, 1994). The estimates $(\hat{s}^2, \hat{H})$ are $l^{1/2}$ -consistent and jointly asymptotically Gaussian, where $l = n/p$, as l, p both tend to infinity, provided that in the likelihood m is replaced by a $n^{(1-H)}$ -consistent estimate of m (Beran, 1994). The following further assumption is made.

Assumption (iii). $l^{1/2}/n^{(1-H)}$ tends to infinity as l, p both tend to infinity.

On the basis of the above remarks, the following plug-in estimator of (1) is proposed:

$$\hat{A} = \frac{\sqrt{2p}}{\hat{s}} \Psi(-\hat{m}, \hat{s}) \exp \left\{ -\frac{1}{2\hat{s}^2} (1-\hat{H})^{-2} \left(-\frac{\hat{m}(1-\hat{H})}{\hat{H}} \right)^{2\hat{H}} K^{2(1-\hat{H})} \right\} I_{(\hat{m} < 0)} + I_{(\hat{m} > 0)} \quad (3)$$

Proposition 1. Under (i)-(iii), $\hat{A}$ converges in probability to A as n goes to infinity.

Our main goal is to construct confidence intervals for (1) that work without the stability condition $m < 0$. Apropos of this, a basic role is played by next proposition.

Proposition 2. Let $c(m, s^2, H) = d A(m, s^2, H) / d m$. Under (i)-(iii), the following hold.

- (a) $m < 0$: $n^{1-H}(\hat{A} - A) \xrightarrow{d} N(0, c(m, H, s^2)^2 s^2)$, $n \rightarrow \infty$
(b) $m = 0$: $\lim_{n \rightarrow \infty} P(n^{1-H}(\hat{A} - 1) \leq x) = 1$ if $x \geq 0$ $\Phi(x / (c(0, H, s^2)s))$ if $x < 0$
(c) $m > 0$: $n^{1-H}(\hat{A} - 1) \xrightarrow{d}$ variate degenerate at 0, $n \rightarrow \infty$

Proposition 2 cannot be directly used to determine confidence intervals for A , since H and the asymptotic variance of $\hat{A}$ are both unknown. The following result holds.

Proposition 3. Under (i)-(iii), (a), (b) and (c) hold true replacing H by $\hat{H}$ and the asymptotic variance of $\hat{A}$ by the plug-in estimate $\hat{v} = c(\hat{m}, \hat{H}, \hat{s}^2) \hat{s}$. The confidence interval for A ($z_{a/2}$ is the $a/2$ th quantile of the normal standard distribution) follows:

$$\hat{A} \pm n^{\hat{H}-1} z_{a/2} \hat{v} \quad (4)$$

Proposition 4. The asymptotic size of the interval (4) is $1-a$ if $m < 0$; $1-a/2$ if $m = 0$; 1 if $m > 0$.

The ATM network considered is the experimental European ATM network. The applications considered use IP over ATM and are videoconference, teleteaching, transport of routing information between IP routers. The time interval (time slot) corresponds to a cell-time. The input process A_n is obtained as the number of cells requiring transmission in the same time slot. The service time process B_n is constant (equal to 1). The sample size is $n = 80000$. The aggregated Whittle's approximated maximum likelihood estimators $\hat{s}^2, \hat{H}$ show that H differs significantly from 0. Confidence intervals for A are shown in Table 1.

$\hat{r}$	K=100	K=500
0.2	$(6.*10^{-17} \pm 3.*10^{-17})$	$(2.2*10^{-50} \pm 1.2*10^{-50})$
0.5	$(3.*10^{-2} \pm 9.*10^{-3})$	$(9.*10^{-5} \pm 1.2*10^{-5})$
0.78	$(3.*10^{-1} \pm 1.5*10^{-1})$	$(8.*10^{-2} \pm 2.*10^{-2})$

Table 1: 95% confidence intervals for $\hat{A}$

As a conclusion, the presence of long range dependence does have a negative impact on the (estimated) overflow probability. The smaller the H , the smaller the A (formula (1)).

References

- Addie R.G. (1999) On weak convergence of long range dependent traffic processes, *Journal of Statistical Planning and Inference*, 80, 155-171.
Beneš V.E. (1962) *General Stochastic Processes in the Theory of Queues*, Addison-Wesley.
Beran J. (1994) *Statistics for Long-Memory Processes*, Chapman and Hall, London.