

Identification of Long Memory in GARCH Models

Identificazione della Memoria Lunga in Modelli GARCH

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Riassunto: questo lavoro estende i risultati di Bollerslev e Mikkelsen (1996) relativi all'utilizzo di criteri di informazione e test per discriminare tra modelli GARCH e FIGARCH. I nostri sviluppi riguardano sia un aumento nella tipologia dei modelli generatori utilizzati sia il numero di alternative considerate in fase di stima. Sono stati esaminati quattro diversi criteri di informazione (Akaike, Hannan-Quinn, Shibata, Schwarz) ed i test di Jarque-Bera, Box-Pierce ed Engle per verificare normalità, autocorrelazione ed eventuali effetti ARCH nei residui standardizzati. Il nostro studio, effettuato tramite simulazioni Montecarlo, evidenzia una buona capacità dei criteri di informazione nell'individuare la presenza di memoria lunga nella varianza condizionale, mentre i test dimostrano di non avere potenza nel distinguere tra i vari modelli.

Keywords: FIGARCH, long memory.

1. Motivation and structure of the analysis

This paper extends the well-known works of Baillie, Bollerslev and Mikkelsen (1996) [thereafter BBM] and Bollerslev and Mikkelsen (1996) [thereafter BM]. In these studies the authors introduced, as a generalisation of the IGARCH, the FIGARCH model, defined as follows:

$$y_t = z_t \sigma_t \quad z_t \sim iid \ N(0,1) \quad (1)$$

$$[1 - \beta(L)] \sigma_t^2 = \omega + [1 - \beta(L) - \phi(L)(1 - L)^d] \varepsilon_t^2 \quad (2)$$

$$\phi(L) = \sum_{j=1}^q \phi_j L^j \quad \beta(L) = \sum_{i=1}^p \beta_i L^i \quad (3)$$

where y_t may represent a series of observed returns, and all parameters are required to satisfy a group of restrictions to ensure the positivity of the conditional variances. This model is assumed to capture the persistence in conditional volatilities, a common finding in financial markets, see among others, Baillie (1996). BBM (1996) provide a Montecarlo analysis showing the consistence of the quasi-maximum likelihood estimators (QMLE), they also give some insight on stationarity and on the asymptotic properties referring to the proof of Lee and Hansen (1994). BM (1996) conduct some limited Montecarlo analysis to assess the ability of information criteria in discerning between different models fitted on a long memory GARCH data generating process

(DGP). All these analyses are performed only on one DGP, the FIGARCH(1,d,0) in view of the fact that they were following Lee and Hansen (1994) and assuming that only in this case the QMLE were consistent and asymptotically normal. In this paper we first show that we cannot apply Lee and Hansen proof to any FIGARCH specification, then we extend, via Montecarlo techniques, the results of BM (1996) in two different directions: first, we consider a wider group of DGPs, specifically GARCH(1,1), FIGARCH(1,d,1), FIGARCH(1,d,0) and FIGARCH(0,d,0), running for each of these models 1,000 replications for three sample lengths (500, 1000 and 2000 observations) and on each simulated series we fit five different models, three long memory GARCH (the three DGPs), a GARCH(1,1) and an IGARCH(1,1); second, we test the ability of four information criteria, Akaike, Hannan-Quinn, Schwarz and Shibata, and then we run tests for normality, correlation and ARCH effects on the standardised residuals of the different fitted models. Even if consistency and asymptotic normality are not directly proved, the Montecarlo simulation in BBM shows how these properties hold for the FIGARCH(1,d,0). Our analysis by the fast convergence of our estimation algorithm together with a limited study on estimates, let us conjecture that consistency is achieved also in FIGARCH(1,d,1) and FIGARCH(0,d,0), however we leave a rigorous proof of asymptotic properties for future researches.

2. Montecarlo results

The main result of this analysis comes from model selection based on information criteria: we verify how each of the four criteria considered, discriminate between long and short memory data generating processes, with peaks up to the 99% of correct identification. However, once long memory is recognized, the four criteria cannot noticeably identify the correct orders p and q , showing, therefore, high variability: as an example consider the FIGARCH(1,d,1) DGP in table 2, in this case the highest frequency of selection is on the FIGARCH(1,d,0) specification. This behaviour depends both on parameters value and on the sample length. Observing tables 1 and 2 we can note that the frequency of selection has a growing mass on the correct generator, consequently, increasing the sample length. This led us to believe that identification problems among different long-memory specifications can be solved or at least reduced by using longer series.

Referring to the other tests, they are not powerful in discriminating among short and long memory, as we can see from table 3. In all the considered tests the null hypothesis of normality, no autocorrelation and no ARCH effects on the standardized residuals, are most of the time accepted independently of the generator and of the fitted model.

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Table 1: Frequency of model selection – DGP: FIGARCH(1,d,0) – $d=0.8$ $\beta=0.5$

	T	I	II	III	IV	V
AIC	500	0,004	0,253	0,017	0,137	0,589
	1000	0,027	0,557	0,002	0,124	0,290
	2000	0,073	0,799	0,000	0,059	0,069
HQ	500	0,000	0,111	0,036	0,049	0,804
	1000	0,000	0,360	0,010	0,046	0,584
	2000	0,009	0,732	0,000	0,036	0,223
BIC	500	0,091	0,403	0,003	0,241	0,262
	1000	0,169	0,602	0,000	0,158	0,071
	2000	0,281	0,640	0,000	0,065	0,014
SH	500	0,005	0,255	0,017	0,138	0,850
	1000	0,027	0,559	0,002	0,125	0,287
	2000	0,073	0,799	0,000	0,059	0,069

T represents sample length; columns from I to V contain selection frequencies respectively of FIGARCH(1,d,1), FIGARCH(1,d,0), FIGARCH(0,d,0), GARCH(1,1), IGARCH(1,1); results are based on 1000 replications

Table 2: Frequency of model selection – DGP: FIGARCH(1,d,1) – $d=0.8$ $\beta=0.5$ $\phi=0.3$

	T	I	II	III	IV	V
AIC	500	0.005	0.228	0.468	0.054	0.245
	1000	0.068	0.501	0.329	0.046	0.056
	2000	0.251	0.553	0.180	0.011	0.005
HQ	500	0.000	0.073	0.573	0.011	0.343
	1000	0.001	0.312	0.529	0.022	0.136
	2000	0.036	0.572	0.368	0.008	0.016
BIC	500	0.230	0.339	0.222	0.124	0.085
	1000	0.452	0.351	0.144	0.048	0.005
	2000	0.660	0.272	0.058	0.009	0.001
SH	500	0.005	0.232	0.467	0.055	0.241
	1000	0.069	0.500	0.329	0.046	0.056
	2000	0.254	0.552	0.178	0.011	0.005

T represents sample length; columns from I to V contain selection frequencies respectively of FIGARCH(1,d,1), FIGARCH(1,d,0), FIGARCH(0,d,0), GARCH(1,1), IGARCH(1,1); results are based on 1000 replications

Table 3: frequency of accepting H_0 at the 1% - DGP: FIGARCH(1,d,0) – d=0.8 $\beta=0.5$

	I	II	III	IV	V
Q(5)	0,905	0,955	0,971	0,954	0,977
Q(10)	0,957	0,968	0,977	0,970	0,978
Q(20)	0,966	0,974	0,688	0,974	0,740
Q(50)	0,981	0,986	0,985	0,984	0,990
Q(100)	0,985	0,986	0,986	0,984	0,986
$Q^2(5)$	0,958	0,978	0,028	0,968	0,983
$Q^2(10)$	0,980	0,990	0,025	0,981	0,985
$Q^2(20)$	0,988	0,989	0,003	0,984	0,714
$Q^2(50)$	0,987	0,991	0,212	0,980	0,985
$Q^2(100)$	0,988	0,988	0,367	0,973	0,979
JB	0,887	0,885	0,648	0,863	0,865
LM(2)	0,992	0,992	0,992	0,989	0,989
LM(5)	0,988	0,988	0,985	0,987	0,987
LM(10)	0,988	0,988	0,986	0,988	0,988
LM(20)	0,989	0,989	0,986	0,990	0,990
LM(50)	0,993	0,994	0,990	0,993	0,993

$Q(n)$ stands for the Box-Pierce statistic computed on standardized residuals, where n is the number of correlations considered in the test; $Q^2(n)$ is computed on squared standardized residuals; JB is the Jarque-Bera normality test; LM(n) is the Engle Lagrange multiplier test for ARCH effects, where n is the number of lags considered in the test; columns from I to V show the frequencies of accepting the null hypothesis of the different tests, respectively for FIGARCH(1,d,1), FIGARCH(1,d,0), FIGARCH(0,d,0), GARCH(1,1), IGARCH(1,1); results are based on 1000 replications; sample length is fixed at $T=2000$