

The use of prices in the DM^2 complete demand system¹

Il sistema completo di domanda DM^2 con prezzi variabili

Gustavo De Santis
Università di Messina,
V. Cannizzaro 9, 98122 ME,
desantisg@multipolmail.unime.it

Mauro Maltagliati
Dip. di Statistica "G.Parenti",
V.le Morgagni, 59, 57134 FI,
mmaltag@ds.unifi.it

Riassunto: Nella stima di sistemi di domanda di beni di consumo, l'introduzione di variabili demografiche si rivela spesso indispensabile. Tra i modi di inserimento, quello che noi proponiamo, il DM^2 (*Decomposition Model of the effects of Demographic Metamorphosis*), appare particolarmente indicato, sotto il profilo sia della teoria microeconomica sia della bontà di adattamento del modello ai dati empirici, e questo in analisi tanto di un singolo anno (a prezzi fissi) che di più anni (in presenza di prezzi variabili).

Keywords: Equivalence scales; complete demand system; cost and style effects.

1. Introduction

Let us assume that, within households, utility, i.e. the consumer's satisfaction an increasing function of per-capita-adjusted consumed quantities - is the same for all members, and let the vector ${}^h\mathbf{a}$ synthesise the differences between households, like location, age of members, sex, etc. (${}^h\mathbf{a}=[0]$ for the reference household).

In this context, one of the most interesting question is: what can we say about utility when both resources and household characteristics change? What relative increase in (minimum) costs must the reference household incur, if, for instance, the number of its members increases, but utility must be preserved at the same level (U_0)?

To this long-debated issue, dating back to the times of Engel, at the end of the 19th century, we contributed recently (Maltagliati and De Santis, 2001), with a new approach called DM^2 - *Decomposition Model of the effects of Demographic Metamorphosis*. In essence, we suggest that the problem can be solved by estimating at the same time both a relative cost (or equivalence scale) ${}^h\epsilon$ and a set of "demand shifters" ${}^h s_i$, called style or taste effects, one for each of the I consumption items in which total expenditure may be broken down. In the simplest possible version, our approach reduces to

$$1) \quad {}^h w_i = \mathbf{a}_i + {}^h s_i + \mathbf{b}_i \left(\log \frac{{}^h X - \sum p_i {}^h s_i}{{}^h e} \right) = \mathbf{a}_i + {}^h s_i + \mathbf{b}_i \left(\log \frac{{}^h X}{{}^h e} \right)$$

¹ Luigi Cannari's critics on an earlier draft are gratefully acknowledged, and so is financial support from the Italian Ministry of Education, University and Scientific Research. Processing of Istat (Italian Institute of Statistics) microdata on Household Budgets falls entirely under the authors' responsibility.

where w_i is the consumption share of item i ; $\mathbf{p}=[p_i]$ is a vector of prices; X represents the income (or total expenditure) of household h ; ${}^h s_i [= s_i({}^h a)]$ and ε are the style and the cost effect we mentioned above; and α_i , β_i and s_i are parameters to be estimated, subject to the summing-up constraints $\sum \alpha_i=1$, and $\sum \beta_i=\sum s_i=0$.

This paper is a follow-up of our previous work, and it sets out to show that our approach 1) is not only empirically useful, but also theoretically (i.e. micro-economically) well-founded; and 2) can be used in a multi-year setting, i.e. when relative prices vary - although its main merits emerge in a single-year context, with fixed prices.

2. Micro-economic foundations

Let us assume that utility is an increasing function of per-capita-adjusted consumption, for instance *à la* Gorman, with extra members needing some fixed input (food, clothes, ...; *translating*, with $\tau_i \geq 0$), generally interpreted as a subsistence minimum, and, on top of this, that they also impose some proportional increase in at least some of the inputs (*scaling*, with $\sigma_i \geq 1$)

$$2) {}^h U = U \left(\frac{{}^h q_i - t_i}{s_i} \right) \Rightarrow {}^h U = {}^r U \Leftrightarrow {}^h \underline{q}_i = q_i {}^h \sigma_i + {}^h \tau_i$$

where q_i ($= {}^r q_i$) denotes the quantities consumed by the reference household r , and the ${}^h \underline{q}_i$ are underlined if they grant household h precisely the same utility as household r . For any given set of prices $\mathbf{p}$, equation (2) implies that household h will be as well off as household r only if its global expenses ${}^h X$ exceed those of the reference household ${}^r X$ by a scalar ${}^h \varepsilon$, or simply ε , given by

$$3) e = \frac{{}^h X(U_0, \mathbf{p}, {}^h \mathbf{a})}{{}^r X(U_0, \mathbf{p}, {}^r \mathbf{a})} = \frac{\sum_i p_i q_i s_i + \sum_i p_i t_i}{\sum_i p_i q_i}$$

With a few algebraic passages - i.e., defining ${}^h s_i = s_i = q_i \sigma_i + \tau_i - q_i \varepsilon$; multiplying both sides of eq. (2) by prices p_i , and dividing them both by ${}^h X$ ($= \varepsilon {}^r X$) -, it can be shown that the “equivalent” (or constant-utility) budget shares are

$$4) {}^h \underline{w}_i = w_i + p_i s_i / {}^h X$$

where the summing-up constraint ($\sum {}^h \underline{w}_i = \sum w_i = 1$) holds, because the s_i 's are but a difference from the mean relative cost ε defined in (3), so that $\sum p_i s_i = 0$, regardless of the price system $\mathbf{p}$. In short, our approach is based on a *direct* utility function like (2), but leads to an *indirect* utility function of the type

$$5) {}^h y(\mathbf{p}, {}^h X, {}^h \mathbf{a}) = {}^r y \left(\mathbf{p}, \frac{{}^h X - \sum p_i s_i}{e}, {}^r \mathbf{a} \right)$$

where the “demographic” parameters, ε and s_i , focus on what we deem to be the most relevant consequences of the interaction between the utility function in (2) and prices $\mathbf{p}$: the relative cost ε of any household characteristic $^h\mathbf{a}$, and, beyond that, the modification in the consumption profile that it causes, s_i (cf. eq. 4), with some goods being consumed proportionally less (if $s_i < 0$), and others proportionally more (if $s_i > 0$) than in the reference household.

Note, in passing, that eq. (5) holds for *any* type of utility function, not necessarily of the kind defined in (2), subject to the sole condition that ε be *the* equivalence scale, e.i. the ratio between the minimum expenses required to reach a pre-defined utility level in two different households, r and h , as in equation (3).

3. Extension to several years

The analysis has thus far been confined to just one year, and it is worth noticing that this constitutes a remarkable improvement over what is normally possible with complete demand systems, which are either too simplistic, e.g. Ray’s (1983) restricted price scaling, or impossible to estimate on a single cross section.

Over several years, our system of “demographic extension” DM^2 works equally well with all kinds of complete demand system. With Deaton and Muellbauer’s (1980) *AI*, or *Almost Ideal Demand System*, for instance, consumption shares are defined as

$$6) w_i(\mathbf{p}, X) = a_i + \sum_j g_{ij} \log p_j + b_i \log \frac{X}{P}$$

where P is a convenient price index (for instance, such that $\log P = \sum w_k \log p_k$) and where the γ -parameters measure how sensitive the consumption of good i is to the price variation of good j . Introducing our demographics in this model leads to

$$7) w_i(\mathbf{p}, ^hX, ^h\mathbf{a}) = a_i + ^hs_i + \sum_j g_{ij} \log p_j + b_i \log \frac{X}{eP}$$

This is the formula we applied to a selection of households (up to 5 members) taken from the Istat yearly Budget Surveys, for the years 1987-1996, and using 8 consumption categories (cf. table 1). As it always happens with the *AI* model, the theoretical condition $\gamma_{ij} = \gamma_{ji}$ ($\forall i, j$), does not materialise, and it must be imposed by the researcher, although this constraint may have relevant consequences on the estimated parameters.

Apart from this, our results confirm that using the DM^2 model constitutes an improvement over all (to us) known alternatives. Statistically, the LL (in thousands) rises from 94.8 in model (6) without demographics, to 95.5 in Ray’s (1983) model with ε only, to 99.1 in our model (7) with both ε and s_i . And, economically, we can attribute variations in the consumption profile of different households partly to costs and partly to style. If costs are set to 1 for a one-person households, each additional individual depresses utility so much, that restoring it to its previous level would in principle require extra resources of .53. This estimate falls between the extremes one obtains using Ray’s (17%) or Engel’s model (67%; Carbonaro, 1986), and this conforms to

expectations, because these models were suspected to lead to under and over-estimates, respectively.

Besides, we get evidence of the long suspected (but never measured) fact that, for *any given level of utility*, smaller households (i.e. prevalently elderly persons) prefer to spend relatively more on housing, food and health, but less on apparel, transport, etc.

Table 1. *Synthetic results of models (7) and (8), Italy 1987-96 (*)*

Model		(7)	(8), no style	(8)
# of parameters		42	43	50
Log-Likelihood		94 807	95 470	99 142
Cost		ε	17.3% (32.6)	53.1% (38.4)
Style effects on:	food	s(1)	-- (--)	-0.27% -(4.4)
	apparel	s(2)	-- (--)	0.62% (30.6)
	housing	s(3)	-- (--)	-3.61% -(86.5)
	furniture	s(4)	-- (--)	0.08% (5.0)
	health	s(5)	-- (--)	-0.16% -(17.4)
	transport	s(6)	-- (--)	2.15% (48.4)
	education	s(7)	-- (--)	0.59% (30.2)
	other	s(8)	-- (--)	0.59% (--)

(*) On 5000 cell means: 10 years x 5 hh dimensions x 100 expenditure percentiles. (t-values)

4. Conclusions

All demographic modifications cause both a “taste” and a “cost” effect, that need to be analysed separately. Both dimensions depend on several potentially relevant variables like region of residence, and, for each household member, age, sex, education, etc. Unfortunately, because of cross-products, the estimation load in multi-year complete-demand systems soon exceeds the capabilities of most PCs. On top of this, these models, if unconstrained, frequently lead to inconsistent results, and if constrained may produce biased estimates. Thus far, the only available alternative was the use of demand systems either incomplete (e.g. Engel) or complete, but far too simple and therefore inadequate (e.g. Ray’s restricted price scaling). The DM^2 model seems to strike a satisfactory balance between these opposing requirements, and can be used both in multi-year and (preferably, in our opinion) in single year, fixed-price settings.

References

- Carbonaro G. (1986) L'ineguaglianza in Italia dal 1973 al 1981, in *Studi in onore di Silvio Vianelli*, Istituto di Statistica, Facoltà di Economia, Palermo, 881-924.
- Deaton A., Muellbauer J. (1980) An almost ideal demand system, *American Economic Review*, 70 (2), 312-326.
- Maltagliati M., De Santis G. (2001) De gustibus non est disputandum? A new approach to the estimation of equivalence scales, *Statistical methods and applications*, 10 (1).
- Ray R. (1983) Measuring the cost of children. An alternative approach, *Journal of Public Economics*, 22 (1), 89-102.