

Reliability of a University Course. The Evaluation through the Quantitative Fault Tree Analysis (FTA)

Affidabilità di un Corso Universitario. Valutazione tramite la Fault Tree Analysis Quantitativa

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Riassunto: in questo lavoro viene proposta la FTA quantitativa per valutare il Corso di Laurea in Economia e Commercio dell'Università di Firenze. La FTA richiede la costruzione di un diagramma logico, detto albero degli eventi, che rappresenti il sistema sotto analisi. Successivamente, viene valutata l'affidabilità del sistema calcolando la probabilità del *top-event*, ovvero l'evento che causa il guasto totale del sistema. Nel nostro caso, il *top-event* è definito come: "non sostenere i sei esami fondamentali del primo anno entro la durata legale del corso (quattro anni)".

Keywords: Reliability, Reparable System, Fault, *Fault Tree Analysis*.

1. Introduction

Recently, the methods for the assessment of academic institutions have been improved following two directions: the measurement of external effectiveness (like the probability to work after graduation); the measurement of internal performance through students' satisfaction towards the quality of teaching and educational facilities provided. Minor efforts have been devoted to evaluate the performance in terms of reliability. This paper is concerned with the application of the *Fault Tree Analysis* to measure the reliability of the Course in Economics and Business Administration (University of Florence). We used the career data of two cohorts of students recruited in 1990-91 and 1991-92.

The FTA is a reliability method for a reparable system to describe the states of the system's components (basic events) and the connections between these basic events and the system's state (top-event). We take into account the six basic exams of the first year, which represent the basic events of our simplified system: *Economics (ECO)*, *Civil Law (CL)*, *Public Law (PL)*, *Statistics (STA)*, *Accounting (ACC)*, *Mathematics (MAT)*. The top-event is identified as the non-completion of the six exams within the established duration of the course (four years). The problems discussed in the following sections are related to the possibility of the application of FTA in this area.

2. Theory

The essential element of the FTA technique is a logic diagram that displays the interrelationships between a potential critical event (top-event), which represents the

system failure, and the reasons for this event such as the failure of a specific component (basic event). The connections between the basic events and the critical event are represented by means of logic gates. The output from a logic gate is determined by the input events. The final result is called fault tree; it describes the dynamic change of system states when components fail. The main logic gates applied in FTA are: OR-gate and AND-gate (Figure 1). A fault tree with a single AND-gate represents a parallel system, where the top-event occurs if all the input events occur. A fault tree with a single OR-gate represents a series system, where the top-event occurs if at least one of the input events occurs.

Let us now analyse a structure that is modelled as a fault tree. Consider the state variable y_i related to the basic event i ($i=1, \dots, n$), that takes value 1 when the basic event occurs, 0 otherwise. The vector $y: \{y_1, \dots, y_n\}$ is the state vector of the system. In addition, we define the structure function $y(y)$ that describes the top-event state; it assumes value 1 when the top-event occurs, 0 otherwise. Let $q_i = P(y_i=1)$ be the probability that the basic event i occurs and causes the failure of the component i . Let $q_0 = P(y(y)=1)$ be the probability that the top-event occurs; q_0 measures the system unreliability and is the final result of the quantitative FTA.

Under the assumption of independence among basic events, the function $y(y)$ and the probability q_0 for a fault tree with a single AND-gate (parallel system) are, respectively:

$$y(y) = \prod_{i=1}^n y_i \quad q_0 = \prod_{i=1}^n q_i \quad (1)$$

$y(y)$ and q_0 for a fault tree with a single OR-gate (series system) are:

$$y(y) = 1 - \prod_{i=1}^n (1 - y_i) \quad q_0 = 1 - \prod_{i=1}^n (1 - q_i) \quad (2)$$

In a number of situations, the analysis based on the assumption of independence will lead to unrealistic results. If we consider a set of basic events positively correlated⁽¹⁾, expression in (1) gives an under-estimation of q_0 for a parallel system, (2) gives an over-estimation of q_0 for a series system. More precisely, in the case of a series system, we have:

$$q_0 \leq 1 - \prod_{i=1}^n (1 - q_i) \quad (3)$$

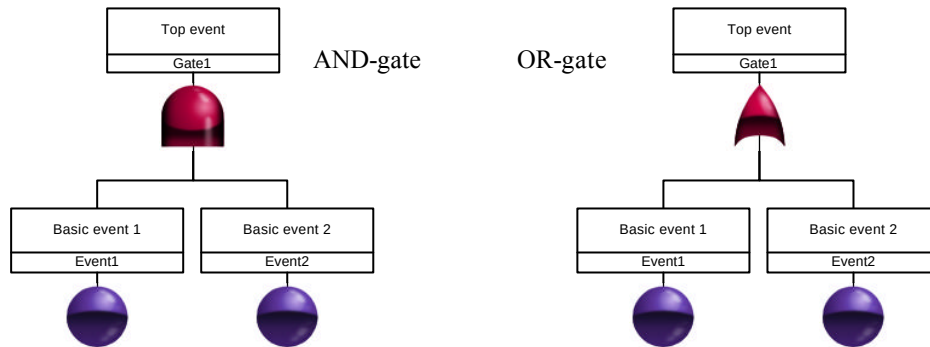
Since the union of a finite number of events includes each single event, a lower bound for q_0 in a series system can be obtained by considering that:

$$q_0 \geq \max_i q_i \quad (4)$$

Hence, an estimate of q_0 (even though approximate) for a series system, can be obtained through the geometric mean of the bounds in (3), (4). This is the ‘square root method’ described in Høyland e Rausand (1994; section 8.4).

⁽¹⁾ A positive dependence occurs when the fault of a component increases the likelihood of a fault of another component (Høyland e Rausand, 1994; Chp. 8). For two events A and B we define as positive dependence the relation: $P(A|B) \geq P(A)$. With two binary variables, as the functions described here, a positive association is equivalent to a positive correlation.

Figure 1: Fault Tree examples



3. FTA and the University Course: Theoretical Aspects

In this section, the use of FTA to evaluate the reliability of a university course is approached from a critical point of view, following the usual steps of FTA in the technological area (step1-4).

1) System definition

In order to derive the structure of our system (that is not a physical system), we consider the syllabus of the student. It is formally established to obtain the degree and it explains the set of the exams to be passed. In this paper, we turn our attention to the six basic exams of the first year of the Course in Economics and Business Administration (see section 1).

2) Top-event definition

The top-event can be expressed as *non-completion of the six exams within four years*. The system so obtained is composed of $n=6$ series events, and each basic event can be defined as “failing the i -th exam within four years”, $i=1,\dots,6$.

3) Construction of the Fault Tree

The fault tree is composed of six basic events and a single OR-gate. At this step of the analysis, the main problems are related with the computation of the reliability parameters (values) of the components, which are unknown. Their estimation is obtained by analysing student's careers. However, student's performance depends also on his/her individual characteristics; hence, the estimation of q_i through the *proportion of students which fail the i -th exam within four years*, is not satisfactory. For evaluating the reliability of the course, two possible solutions could be: (1) the computation of q_0 for a *representative* category of students; (2) the computation of a specific weight quantifying student's ability, to derive the following measure of q_i :

$$\frac{\text{weighted number of students failing the } i\text{-th exam within 4 years}}{\text{weighted total number of students}}$$

Since we are evaluating the components' unreliability, smaller weights should be attached to less able students. In this paper, solution (1) is considered by stratifying students with respect to their final outcome.

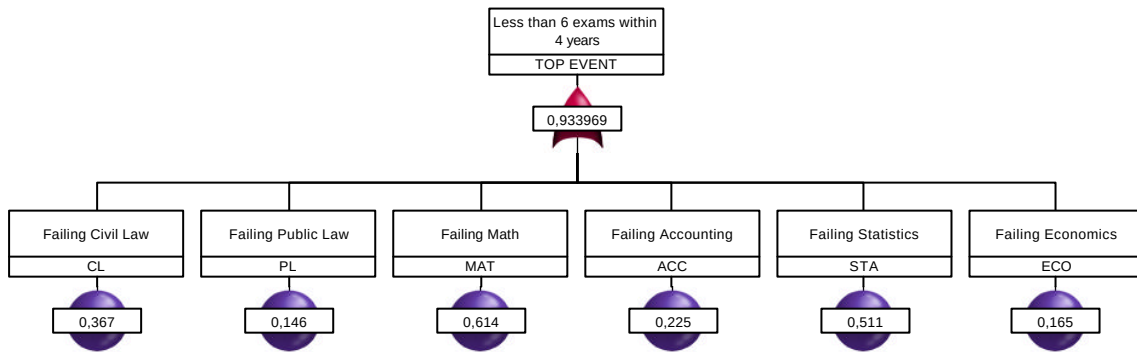
(i) Tree analysis. The assumption of independence among basic events is untenable; probably, the nature of the basic events determinates a positive association. Hence, upper and lower bounds for q_0 are computed.

4. Empirical Analysis: Preliminary Results

The application examines only the “regular” students, namely the set of students who passed, at least, one exam of the first year ($n=1587$). The two cohorts, described in section 1, are merged to build a larger data-set. Only 354 students out of 1587 (22.3%) passed the six exams within four years. The six basic events are positively associated; namely, the binary correlation coefficient ranges from 0.2315 (MAT,ACC) up to 0.4452 (CL,ACC).

Figure 2 represents the *fault tree* with the probabilities q_i obtained by the relative frequency of failures and q_0 computed by formula (2). The top-event probability is notably over-estimated, regarding that the correspondent observed value is 0.777. The lower bound is 0.614 (formula (4)), and q_0 , estimated through the square root method, is equal to 0.757.

Figure 2: Fault Tree for the case study



In order to reduce heterogeneity among subjects, we stratified them conditioning to similar characteristics (not degree, degree and score), assumed as proxy variables of the ability of the student (Table 1). Through stratification and the use of the square root method, we get q_0 estimates which are systematically smaller than the current values. By assuming that q_0 is adequately estimated for the category ‘score>105’, a 10% decrease of MAT unreliability determinates a 6,8% decrease in system unreliability.

Table 1: system’s and basic components’ unreliability (at 1999/12/31)

Outcome, score	Components unreliability q_i						System unreliability q_0				Total n.
	CL	PL	MAT	ACC	STA	ECO	Lower bound	Upper bound	Current value	Estim. (*)	
No degree	0.576	0.245	0.840	0.367	0.738	0.291	0.840	0.994	0.963	0.914	805
≤100	0.209	0.074	0.484	0.115	0.373	0.061	0.484	0.803	0.738	0.623	244
101-105	0.151	0.029	0.395	0.054	0.283	0.024	0.395	0.670	0.624	0.514	205
>105	0.096	0.030	0.300	0.069	0.204	0.024	0.300	0.556	0.450	0.408	333

(*) square root method

References

Høyland A., Rausand M. (1994) *System Reliability Theory*, Wiley & Sons, New York.