

Dynamic Conditional Correlations: Some Critical Remarks

Dynamic Conditional Correlations: Qualche Considerazione Critica

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Riassunto: In questo lavoro si richiamano alcune delle caratteristiche del modello DCC suggerito da Engle (2002a) mettendo in risalto come alcune proprietà delle varianze stimate richiedano ulteriori approfondimenti. Si suggerisce di usare una diagnostica sulla stima della varianza condizionata dei residui standardizzati, di procedere ad una distinzione degli effetti di asimmetria nella varianza condizionata da quelli nella correlazione condizionata, dato che una cattiva specificazione dei primi ha serie conseguenze sulla stima dei secondi e, infine di considerare una specificazione che consideri gli effetti della dipendenza della correlazione condizionata da stati latenti.

Keywords: GARCH Models, Multivariate Distributions, Conditional Correlations

1. Introduction

The award of the 2003 Nobel Prize for Economics to Robert Engle gave a strong recognition to a line of research aimed at modelling the second moments of a stochastic process. The twenty-year old literature on GARCH modelling has evolved from the original model which stated an important distinction between models for conditional and unconditional distributions to extensions which have been motivated with the need to reproduce the empirical regularities exhibited by financial time series and to produce conditional variance estimates and forecasts to be used in risk management, asset allocation and pricing financial derivatives. From a statistical point of view, new contributions have provided better estimators or have investigated more desirable properties of the processes themselves (stationarity conditions, existence of moments, asymptotic properties, and so on).

The new challenges to this body of literature have been outlined by Engle (2002b): next to high frequency volatility models, option pricing and hedging, Multiplicative Error Models (Engle and Gallo (2003)) and simulation methods for conditional expectations, a primary role is played by multivariate models. The main motivation comes from modelling the tradeoff between risk and return: in a number of practical applications (risk management and asset allocation in particular) there is a need to model the conditional covariances (or correlations) between asset returns. The efforts to model second-order mixed moments are quickly met by the curse of dimensionality: with 20 assets the number of parameters to be estimated is 250 with the constant correlation model by Bollerslev (1990); it is 630 with the VEC model (Bollerslev *et al.* (1988)), and it is a staggering 1010 with the BEKK model (Engle and Kroner (1995)).

An alternative route to multivariate conditional covariance modelling has been suggested by Engle himself under the name of Dynamic Conditional Correlation model (henceforth DCC, Engle (2002a)) which applies the GARCH principle of a process which is con-

ditionally autoregressive to modelling the asset return correlations directly and provides a two step estimation procedure which may be applied to a large number of assets as shown by Engle and Sheppard (2001). In what follows I will present the main features of the model, illustrating its mechanism with an empirical example and point out some questions the model is yet unable to answer.

2. The DCC Model

Estimating correlations is of paramount importance in finance: from portfolio management and optimization to hedging, from models of term structure to the study of the behavior of assets during rallies (bull and bear markets), the dynamics of joint distributions can be approximated by second moments (variances and covariances). In the DCC case, the starting point of the model is the consideration that a $n \times 1$ vector of asset returns r_t may be assumed to have a distribution conditional on an information set I_{t-1} with mean zero (the signal in the mean is always very weak at any rate) and a time-varying variance-covariance matrix, namely $r_t|I_{t-1} \sim \mathcal{N}(0, H_t)$. The object of interest is the $n \times n$ matrix H_t which in the present context can be seen as the product $H_t = D_t R_t D_t$ with D_t a diagonal matrix with conditional standard deviations on the diagonal and zeros elsewhere and R_t a time-varying correlation matrix. The main idea is that if one standardizes returns by $\epsilon_t = D_t^{-1} r_t$ then $E(\epsilon_t \epsilon_t' | I_{t-1}) = R_t$. To visualize matters, in the bivariate case, let us consider the mean reverting specification for the bivariate correlation ρ_t

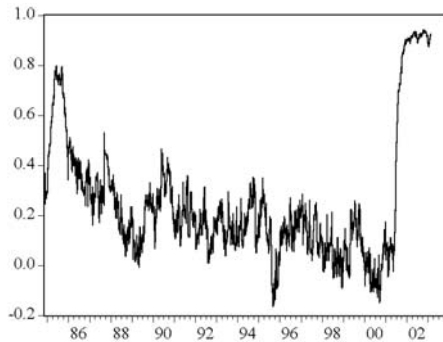
$$\rho_t = \frac{q_{12,t}}{\sqrt{q_{1,t}q_{2,t}}} \quad (1)$$

$$q_{i,t} = 1 + \alpha(\epsilon_{i,t-1}^2 - 1) + \beta(q_{i,t-1} - 1), \quad i = 1, 2 \quad (2)$$

$$q_{12,t} = (1 - \alpha - \beta)\bar{\rho}_{12} + \alpha(\epsilon_{1,t-1}\epsilon_{2,t-1}) + \beta q_{12,t-1} \quad (3)$$

where $\bar{\rho}_{12}$ is the unconditional correlation between assets 1 and 2.

Figure 1. *Conditional Correlation: NASDAQ and S& P500 Indices*



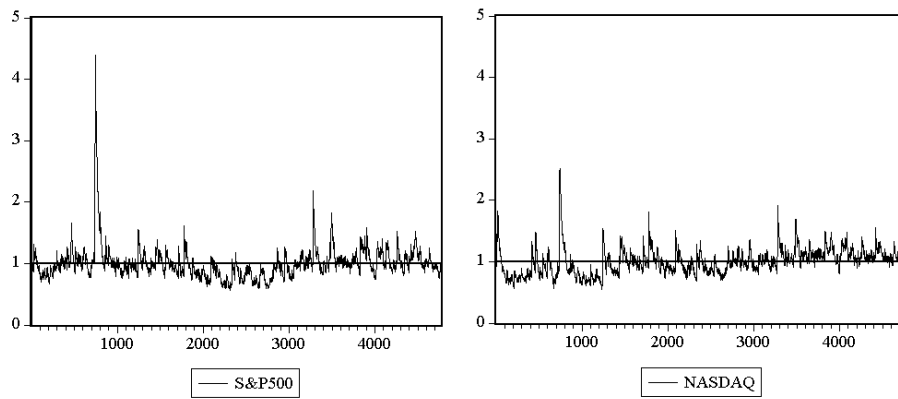
The estimation strategy suggested by Engle (2002a) is a two-step procedure justified by a separability of the likelihood function. In the first step one estimates one GARCH-type univariate models separately on each asset, then maximizes a function which applies (2) and (3) to residuals standardized by estimated conditional standard deviations. As an example, the bivariate dynamic conditional correlation between the Nasdaq 100 and the Standard and Poor's 500 composite indices estimated by the mean reverting DCC (with

Threshold GARCH based conditional variances) over the period Nov.12, 1984 to Oct.3, 2003 is shown in Figure 1. Notice that the trend of the correlation between the two indices which was approaching zero was dramatically interrupted by the events of September 11, after which the correlation shot up toward one.

3. Some Open Questions

1. The estimated conditional variances of the standardized residuals A first issue relates to the time series profile of the conditional variances estimated on the standardized residuals. By looking at Figure 2, it is interesting that there is no strong surge in volatility after Sep. 11, but mostly it is apparent that the two series oscillate around one, but not in a random fashion as a lot of autocorrelation shows up in their behavior (as a matter of fact the first order autocorrelation of either series is close to 0.97). The properties of using these estimated series as a diagnostic tool to infer about the correct specification is a necessary development.

Figure 2. *Estimated Conditional Variances on Standardized Residuals*



2. The asymmetric effects The question of the specification of the model in order to accommodate asymmetric effects becomes here an interesting one, since an asymmetric reaction of the correlations to bad news in both assets is conceptually different from the asymmetric effects which conditional variances are subject to. The generalization of the model seen before has been suggested by Cappiello *et al.* (2003)

$$q_{i,t} = \sigma_i^2 + a_i^2(\epsilon_{i,t-1}^2 - \sigma_i^2) + g_i^2(\epsilon_{i,t-1}^2 \mathbb{I}_i^- - \sigma_i^2) + b_i^2(q_{i,t-1} - \sigma_i^2), \quad i = 1, 2$$

$$q_{12,t} = \bar{\rho}_{12} + a_1 a_2(\epsilon_{1,t-1} \epsilon_{2,t-1} - \bar{\rho}_{12}) + g_1 g_2(\epsilon_{1,t-1} \epsilon_{2,t-1} \mathbb{I}_{12}^- - \bar{\rho}_{12}) + b_1 b_2(q_{12,t-1} - \bar{\rho}_{12})$$

but a bad specification of the asymmetric effects in modelling the univariate conditional variance at the first step is bound to affect negatively the specification of asymmetry in the correlations at the second step. Identification of which asymmetry is present in the data therefore deserves further attention.

3. State-dependent correlations The third issue is connected to the role played by the other assets included in the information set. From earlier discussion, the simple correlation between the two assets does not involve what other assets are being considered. In this respect, the parameterization of the DCC seems to be lacking some substantial link

across assets. Some simple stylized facts should be illustrative of this point. Let us consider two assets (the S&P500 and the NASDAQ over the same sample period as before) and consider the correlation between returns grouped according to whether the univariate GARCH based conditional variance at time $t - 1$ turns out to be lower or higher than the unconditional variance over the whole period. We would have four regimes for which the simple correlations are estimated in the first column of Table 1. It is interesting to observe the asymmetry from low–high to high–low. The need to make estimated correlations state dependent is even clearer when one considers correlations computed by inserting a third asset (the DJ30) and repeating the exercise, this time across the combination of the eight ensuing regimes. The variability of the estimates across the last two columns of Table 1 is even more striking. Similar results are obtained for the MIB30, DAX and CAC40 indices.

Table 1. *State Dependent Correlations*

S&P500	NASDAQ		Dow Jones 30	
States		Simple	Low	High
High	High	0.24	0.04	0.29
Low	High	0.09	0.12	-0.08
High	Low	0.27	0.22	0.29
Low	Low	0.12	0.13	-0.08

In brief, the DCC model has a lot of interesting features which are definitely worthy of attention (parsimonious parameterization in particular), but at the present stage there are a few issues which limit the strength of the conclusions drawable from the model estimation. An alternative, promising strategy seems to be one pursued in the Dynamic Principal Components by Aielli and Gallo (2004) where the parsimonious parameterization is achieved by modeling the time varying covariance matrix through a spectral decomposition.

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