

Dynamic Conditional Correlation Models: Block Structures and Markov Switches for Contagion Analysis

*Modelli con Correlazioni Condizionali Dinamiche:
Strutture a Blocchi e Cambi di Regime
per un'Analisi del Contagio Finanziario*

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Riassunto. A partire dal contributo di Engle (2002), il lavoro introduce ed analizza diverse parametrizzazioni a correlazioni dinamiche (DCC). Dopo un'introduzione generale, che riorganizza gli attuali contributi, sono presentate due possibili estensioni: l'introduzione di una struttura a blocchi nei parametri del modello e l'inclusione di cambi di regime nella correlazione non condizionale del modello. I vari modelli analizzati sono quindi utilizzati per l'analisi del contagio tra mercati azionari. Gli attuali contributi in tema di contagio sono concentrati su metodi non-parametrici: il nostro obiettivo è di derivare validi approcci parametrici per l'analisi dello stesso. Diversi modelli DCC sono quindi utilizzati per valutare la costanza delle correlazioni e della dinamica delle stesse tra indici azionari, di mercati occidentali e del sud est asiatico.

Keywords: dynamic correlation, regime switching, contagion

1. Introduction

Since the seminal work of Bollerslev (1990), multivariate GARCH models attracted considerable interest given their direct application in both financial and economic empirical researches. By now, they represent a fundamental tool for asset management as well as they are included in most analysis concerning financial markets. They have been extended and updated following the enormous literature of the univariate GARCH models, trying to figure out if empirical findings could be implemented even in a multivariate setting. However, a first order of problems came into play when considering large dimension multivariate GARCH models: we refer to the complexity of estimation procedures, which directly derives from the elevated number of parameters. A second set of problems concerns the asymptotic properties of the quasi-maximum likelihood estimators for this type of models, which is not yet theoretically derived. These reasons suggested the introduction of Constant Conditional Correlation models by Bollerslev (1990), where multivariate GARCH structures can be decomposed in a univariate set of GARCH models and in a constant correlation matrix. The obvious

restriction imposed in this setup is not consistent with financial market time series where correlations vary over time. In the last years a new class of model appears, the Dynamic Conditional Correlation, introduced by Engle (2002).

2. Dynamic Conditional Correlation models

For simplicity, we do not specify the model mean, which is simply assumed equal to zero. We have then a set of variables identically distributed with a time dependent variance covariance matrix $E_t \sim iid(0, H_t)$. The variance matrix is then decomposed in the product of a diagonal matrix including time dependent standard deviations, modelised by univariate GARCH models, and a time dependent correlation matrix: $H_t = \text{diag}(\sigma_{it}, j=1 \dots k) R_t \text{diag}(\sigma_{it}, j=1 \dots k) = D_t R_t D_t$. A quadratic structure is imposed on R_t to ensure we are working with a correlation matrix: $R_t = [\text{diag}(q_{11,t} \dots q_{kk,t})]^{-1} Q_t [\text{diag}(q_{11,t} \dots q_{kk,t})]^{-1}$. Finally, $Q_t = (1 - \alpha - \beta) \Gamma + \alpha N_t + \beta Q_{t-1}$, where Γ is the unconditional correlation estimated over the available sample and $N_t = D_{t-1}^{-1} E_{t-1}$ is the vector of the standardised residuals. The DCC model simply includes a GARCH-type structure on correlations. In its first specification, the model assumes the same dynamics for all correlations, which is inconsistent with data, since it seems unrealistic to assume that the correlation between two stocks have the same dynamic structure of the correlations between two bonds. This fact is particularly relevant if we are working with portfolios including both classes of assets. The model could then be generalised replacing the scalars α and β with two matrices A and B : $Q_t = (ii' - A - B) \odot \Gamma + A \odot N_t + B \odot Q_{t-1}$ where ii' is a matrix filled with ones and $\odot$ is the Hadamard product (i.e. the element by element product). However, by this way we get back to one of the problems of standard multivariate GARCH models: too many parameters. Therefore, some restrictions have to be imposed: Franses and Hafner (2003) suggest $A = aa'$ and $B = bb'$, where a and b are two parameter vectors.

Further generalisations of the model include the Asymmetric DCC of Cappiello, Engle and Sheppard (2002) and the GARCC of Chan, Hoti and McAleer (2003). This last contribution has to be evidenced for the derivation of asymptotic properties: stationarity of the model, consistency and asymptotic normality of quasi-maximum likelihood estimators. However, the structure that Chan *et al.* suggest present some inconsistencies. This point can be solved with a simple modification of their assumptions preserving their main results. A further note concerns their recall to a previous result of Jeantheau (1998) in proving consistency of QMLE: this result provides pointwise consistency of the estimators and not a uniform consistency.

One of the objectives of multivariate correlation models is to combine feasibility of the parameter estimation with model flexibility and to provide adequate structures to practitioner needs or to theoretical studies. The following extensions go in this direction.

2.1 Block-DCC

Traditional asset allocation techniques diversify a portfolio among classes of assets (stocks, bonds, cash...), sectors (energy, industrials...) or by geographic areas (America, Europe, Pacific...). However, the original DCC model applied to a sectorial portfolio assumes that the correlation dynamic is constant over assets belonging to

different groups, while the empirical evidence is counterfactual (correlations among assets of different classes show different patterns). In our view, a generalisation of the DCC model is needed, introducing a block structure for the parameter matrices. In this case A and B can be considered as partitioned matrices. As an example consider the matrix A (with $n+m+l=k$ the dimension of the system):

$$A = \begin{bmatrix} i_n i_n' \alpha_1 & i_n i_m' \alpha_4 & i_n i_l' \alpha_5 \\ i_m i_n' \alpha_4 & i_m i_m' \alpha_2 & i_m i_l' \alpha_6 \\ i_l i_n' \alpha_5 & i_l i_m' \alpha_6 & i_l i_l' \alpha_3 \end{bmatrix} \quad (1)$$

2.2 MSM-DCC

In a different framework, for contagion analysis, the interest is in detecting whether crises can be transmitted among markets and can create breaks in the normal interdependence. A break can be considered as a regime switch. Therefore, it seems interesting to include regime switches in correlation modelling. The starting point for this analysis is Pelletier (2003) who focused on Constant Correlation formulations. A regime switch is then included in the unconditional correlation:

$$Q_t = (1 - \alpha - \beta) \Gamma(s) + \alpha N_t + \beta Q_{t-1} \quad (2)$$

In traditional DCC formulations, the unconditional correlation is estimated on a sample basis then the inclusion of regime switches creates a relevant increase in parameter number. To solve this problem we act as in Pelletier (2003) assuming that the unconditional correlation becomes state dependent by an additive factor.

3. Detecting Contagion with DCC

In the literature, there is not a general agreement on the definition of contagion and of interdependence across financial markets. In this work we define contagion as a structural break in the linkages between two markets (a break in the interdependence path). Furthermore, the current literature focuses on non-parametric procedures for detecting contagion (see, among others, Forbes and Rigobon, 2002).

Our setup is rather general in order to assume conditional heteroskedasticity and dynamics in the correlations. The dataset used considers a set of stock market indices observed at the daily level: Dow Jones (US), FTSE 100 (UK), Euro Stoxx 50, Nikkei 225 (Japan), Straits Time (Singapore), Hang Seng (Hong Kong) and KLME Composite (Malaysia). The log-returns are computed and in order to eliminate the noise due to overlapping markets a two-day moving average is calculated. The obtained series are then homogenised, i.e. whenever one of the markets is closed for a holiday, the corresponding observation is deleted in all the dataset. Both corrections were also applied by Forbes and Rigobon (2002). Successively, the conditional heteroskedasticity is filtered out with a simple GARCH(1,1) model in order to concentrate on correlations. The obtained series are then analysed: first, rolling correlations are computed. In this way we check for time dependent correlations (i.e. dynamic structures) and for common

patterns among European/American indices and among Asian indices, as well as in the correlations between the two groups.

In the following step, we estimate DCC-type models (standard DCC and the DCC with block parameter structure) with a rolling sample to analyse a possible dynamics in the coefficients: a jump in a coefficient level could be associated with a change in the normal linkages between markets. Finally MSM-DCC models will be considered and the results compared with the previous estimations.

4. Conclusions

The paper rationalises some of the current findings on dynamic correlation models and provides two new extensions of the models: a DCC model with block structures in the parameters and a DCC model with regime switches. The suggested formulations are then used in a contagion analysis using daily data from European, American and Asian stock market indices.

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