

Models Of Labour Demand With Fixed Costs Of Adjustment: A Generalised Tobit Approach¹

Generalizzazione del modello Tobit per la domanda di lavoro a costi fissi

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Riassunto: In questa nota viene esaminato il modello della domanda di lavoro in presenza di costi fissi di aggiustamento mediante una generalizzazione del modello Tobit con doppio troncamento che tenga conto della eterogeneità non osservabile che caratterizza le decisioni delle imprese.

Keywords: Dynamic labour adjustment, fixed costs, Random Effect Double Censored Tobit.

1. Introduction

Traditional models of factor demand rely upon convex and symmetric adjustment costs: the marginal cost of varying the quantity of an input used in the production process is assumed (i) to increase with the size of the adjustment, and (ii) to be the same for positive and negative changes. Adjustment costs of this shape can generate the partial adjustment dynamics assumed in much empirical work. However, the fortune of this highly restrictive model is due more to analytical convenience than to actual empirical relevance: while convex costs implies frequent changes of small size, in practice micro behavior is often characterised by lumpy adjustment, with rare large changes and many periods of inaction (cf. *e.g.*, Davis and Haltiwanger (1991), Gavosto and Sestito (1993)). Consequently, over the last decade there have been several efforts to explore alternative models. In this note we examine the model of employment adjustment under the hypothesis of fixed costs, show that it can be cast in the form of a Double Censored Random Effect Tobit Model, and derive its likelihood function.

2. Modelling Employment Adjustment with Fixed Costs

Consider a single firm operating with the objective to maximise the discounted expected value of future profits. In presence of fixed costs this implies that the adjustment of labour inputs (L) is carried out if the shadow value of the marginal worker, *i.e.* if the cost of not adjusting (c_{NA}), exceeds in absolute value the fixed cost of the adjustment (c_A), either because hiring costs are smaller than expected revenues net of wage costs or

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firing costs are smaller than expected losses (we are assuming symmetry of the costs in order to simplify notation, but the extension to non-symmetric costs is trivial). Define as target employment (L^*) the labour inputs needed in order to deliver the desired amount of output in standard operating conditions (no extra-time nor labour hoarding), and measure disequilibrium with the difference between non-adjustment and adjustment costs. Under the assumption of static expectations or, equivalently, rational expectations and stationarity of target employment, a sufficiently large disequilibrium in period t will cause an immediate shift to the new target level of employment (Hamermesh and Pfann (1996)):

$$L_{it+s} = \begin{cases} L_{it-1} & \text{if } |c_{At}| > |c_{NA t}| \\ L_{it}^* & \text{else} \end{cases} \quad s = 0, 1, \dots \quad (1)$$

The cost of not adjusting is the present value of the difference of the future streams of net total revenues delivered by L_{it}^* and L_{it-1} workers. Define revenues (R), costs per worker (W) and net revenues $\bar{R} = R - W$. Then, assuming costs and revenues are linear we have that $c_{NA t} = \sum_{s>0} (\frac{1}{1+r})^s \bar{R}(L_{it}^* - L_{it-1}) = \frac{(1+r)}{r} \bar{R}(L_{it}^* - L_{it-1})$, where r is the relevant discount rate, and thus (Hamermesh (1989)):

$$L_{it+s} = \begin{cases} L_{it-1} & \text{if } |L_{it}^* - L_{it-1}| < \Delta_{\min} \\ L_{it}^* & \text{else} \end{cases} \quad s = 0, 1, \dots \quad (2)$$

where now the threshold is $\Delta_{\min} = c_A (\frac{1+r}{r} \bar{R})^{-1}$. For estimation purposes it is convenient to rewrite model (2) in terms of the target relative change in employment, $\Delta^* L_{it} = \frac{L_{it}^* - L_{it-1}}{L_{it-1}}$, which, under the assumption that firm i in period $t-1$ is in equilibrium, will be a function of the changes in a set of explanatory variables typically including output and costs. Dividing through (2) by L_{it-1} , dropping the simplifying assumption of symmetric costs and rearranging further we obtain:

$$\Delta^* L_{it} = x_{it} \beta + \epsilon_{it} \quad (3)$$

$$\Delta L_{it} = \begin{cases} \Delta^* L_{it} & \text{if } \Delta^* L_{it} \leq \theta_i^- \\ 0 & \text{if } \theta_i^- < \Delta^* L_{it} < \theta_i^+ \\ \Delta^* L_{it} & \text{if } \Delta^* L_{it} \geq \theta_i^+ \end{cases} \quad (4)$$

where $\Delta L_{it} = \frac{L_{it} - L_{it-1}}{L_{it-1}}$ is the actual relative change in employment, $x_{it} = (x_{1it}, \dots, x_{kit})$ are k exogenous variables (per-worker return, factor inputs and so on) observed on each firm in $t = 1, \dots, T$, and $\beta = (\beta_1, \dots, \beta_k)$ is the coefficient vector. Model (4) is in fact a Double Censored Tobit Model, with the thresholds (θ^+, θ^-) not known. Indeed, these can be conceived as variables likely to depend upon a large number of factors, both economy-wide (*e.g.*, economic cycle) and firm-specific (*e.g.*, size, technology and state of industrial relations), as in the *threshold regression model* (Dagenais (1975)). A different way to take into account the unobserved heterogeneity is to consider a general Chamberlain-like model (see Wooldridge (2002) for a discussion). Let (a) $\bar{x}_i$ be the vector of the average over time of the determinants, (b) ν_i^- and ν_i^+ be independent idiosyncratic noises satisfying $\nu_i^- | x_i \sim N(0, \sigma_{\nu^-})$, $\nu_i^+ | x_i \sim N(0, \sigma_{\nu^+})$. Then, we can assume $\theta_i^- = \gamma^- \bar{x}_i + \nu_i^- < 0$, $\theta_i^+ = \gamma^+ \bar{x}_i + \nu_i^+ > 0$. If x_{it} contains a time-constant variable to make identification possible we assume that its coefficient in γ^+ and γ^- is zero. Model (4) may be rewritten as a "friction model". The standard form of these class of models (Rosett (1959)) is:

$$y_i = \begin{cases} y_i^* - \alpha_2 & \text{if } y_i^* > \alpha_2 \\ 0 & \text{if } \alpha_1 < y_i^* < \alpha_2 \\ y_i^* - \alpha_1 & \text{if } y_i^* < \alpha_1 \end{cases} \quad (5)$$

where $y_i^* = x_i\beta + \epsilon_i$, and the thresholds $\alpha_1(< 0)$ and $\alpha_2(> 0)$ are known. The log-likelihood function can be found in Maddala (1983). Rearranging (4) according to (5) we obtain an extension of the Random Effect Tobit model in case of double censoring "at random". Then, under assumptions (a) and (b), the model assumes the following expression:

$$\Delta^* L_{it} = x_{it}\beta + \epsilon_{it} \quad (6)$$

$$\Delta L_{it} = \begin{cases} \Delta^* L_{it} - \theta_i^- & \text{if } \Delta^* L_{it} \leq \theta_i^- \\ 0 & \text{if } \theta_i^- < \Delta^* L_{it} < \theta_i^+ \\ \Delta^* L_{it} - \theta_i^+ & \text{if } \Delta^* L_{it} \geq \theta_i^+ \end{cases} \quad (7)$$

where the noise ϵ satisfies $(\epsilon_{it} \mid \nu_i^-, \nu_i^+, x_i) \sim N(0, \sigma_\epsilon)$. The log-likelihood essentially combines the structure of those of Random Effect Tobit and friction models:

$$\log L(\psi \mid \Delta L, x, a_i, b_i) = \sum_{i=1}^n \log F_i \quad (8)$$

where $\psi = (\beta, \gamma^-, \gamma^+, \sigma_\epsilon, \sigma_a, \sigma_b)$ is the vector of parameters of interest,

$$F_i(\Delta L_i / x_i, \nu_i^-, \nu_i^+; \psi) = \int \int \left[\prod_{t=1}^T f_t(\Delta L_{it} \mid x, \nu_i^-, \nu_i^+; \psi) \right] \frac{1}{\sigma_{\nu_i^-}} \phi\left(\frac{\nu_i^-}{\sigma_{\nu_i^-}}\right) \frac{1}{\sigma_b} \phi\left(\frac{\nu_i^+}{\sigma_{\nu_i^+}}\right) d\nu_i^- d\nu_i^+ \quad (9)$$

and

$$f_t(\Delta L_{it} \mid x, \nu_i^-, \nu_i^+; \psi) = \begin{cases} \prod_{\Delta L_{it} < 0} \frac{1}{\sigma_\epsilon} \phi\left(\frac{\Delta L_{it} - x_{it}\beta + \gamma^- \bar{x}_i + \nu_i^+}{\sigma_\epsilon}\right) \\ \prod_{\Delta L_{it} = 0} \left[\Phi\left(\frac{\gamma^+ \bar{x}_i + \nu_i^- - x_{it}\beta}{\sigma_\epsilon}\right) - \Phi\left(\frac{\gamma^- \bar{x}_i + b_i - x_{it}\beta}{\sigma_\epsilon}\right) \right] \\ \prod_{\Delta L_{it} > 0} \frac{1}{\sigma_\epsilon} \phi\left(\frac{\Delta L_{it} - x_{it}\beta + \gamma^+ \bar{x}_i + \nu_i^-}{\sigma_\epsilon}\right) \end{cases} \quad (10)$$

The partial effect can be evaluated at the mean value taking the first derivative of

$$m(\hat{\gamma}^- \bar{x}_i - x_{it}\hat{\beta}, \hat{\sigma}^2) + m(x_{it}\hat{\beta} - \hat{\gamma}^+ \bar{x}_i, \hat{\sigma}^2)$$

where $m(c, \sigma^2) = \Phi(c/\sigma)c + \sigma\phi(c/\sigma)$ with respect to the elements of x .

3. Conclusions

In this note we derived the likelihood function of a model of employment adjustment under the hypothesis of fixed costs cast in the form of a Double Censored Random Effect Tobit Model. The key difference between the two models is the status of the censoring thresholds, which are assumed known in the latter but not in the former (in fact, modelling them as a function of a set of explanatory variables may be an important part of the analysis). Hence, a crucial consequence of adopting the friction model approach is that the thresholds, or rather their expressions in terms of the chosen set of explanatory variables, appear as arguments of the Gaussian distribution function in the likelihood. As there is no reason why the thresholds should be symmetrical or the random noise to be the same for both thresholds (indeed, the opposite is likely to hold), double integrals are involved in the computation of the likelihood and the hypothesis that the idiosyncratic

noises ν_i^- and ν_i^+ are independent turns out to be crucial. Although at a first sight this assumption may appear to be very strong, this may not be the case if we take a vector of explanatory variables general enough to actually explain all the systematic variability of the thresholds across firms. Rather, the computational point about the double integrals can be empirically critical, as in practice maximum likelihood estimates may be difficult to obtain with the large sets of individual data ideally to be used for this type of models. Some preliminary Monte Carlo experiments with standard maximisation techniques conducted under the simplifying hypothesis of symmetric threshold have given interesting results, suggesting for instance that the quality of the ML estimates may depend much more heavily than commonly assumed on the degree of the censoring. Directions for future research include modelling the variances of the ν 's across firms and assessing the performance of alternative estimation strategies, such as indirect inference.

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