

A Rolling Constrained Regression Model for Mutual Fund's Ranking

Un Modello di Regressione Vincolata di tipo Rolling per il Ranking dei Fondi Comuni d'Investimento

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Riassunto: Il lavoro focalizza l'attenzione sulle gestioni *multimanager* dei fondi di fondi, ossia di prodotti del mercato finanziario che investono in fondi comuni relativi ad una specifica categoria di mercato (azionario, obbligazionario, strumenti derivati). L'obiettivo è proporre un metodo per il *ranking* dei fondi comuni appartenenti ad una specifica categoria. A tale scopo si considera un modello di regressione multipla vincolato di tipo *rolling* per stimare la composizione dei portafogli dei fondi comuni e dell'indice di riferimento (*benchmark*), da cui ricorrendo all'Errore Quadratico Medio Integrato (MISE) si valuta la prossimità tra i rendimenti dei portafogli dei singoli fondi ed il portafoglio del benchmark. Ciò consente di individuare lo stile di gestione seguito dal gestore di ogni fondo rispetto ad un benchmark considerato rappresentativo del mercato di riferimento. Si sintetizzano brevemente le possibilità offerte dal modello proposto con riferimento all'analisi di un insieme di fondi azionari italiani operanti nel mercato europeo.

Keywords: Benchmarking, Portfolio Management, Performance Attribution, Constrained Linear Regression, Mean Integrated Squared Error.

1. The Framework

Nowadays, among the range of new financial products offered by investment banks, portfolios composed by Mutual Funds (Funds of Funds) are an interesting opportunity for investors. They are aimed to offer an "optimal" portfolio in the Markovitz sense, i.e. a portfolio offering the maximum return with the minimum risk, and can be divided in two categories:

- 1) *Mutual Funds Individual Portfolios (MFIP)*. The investment bank, instead of investing savings in a single financial stock, use it to invest in a basket of Mutual Funds.
- 2) *Mutual Funds Mixed Portfolios (MFMP)*. MFMP is somewhat similar to MFIP, main difference being that the investment is not for a single investor but it is usually made for a collection of savers. In this case, the bank is able to offer a new product obtained by combining quotas of different mutual funds.

We concentrate our attention on MFMP investing on a single asset class (*monobrand management*). The choice of a single investment category allows to start from a set of quite homogeneous products which can be evaluated on the basis of a common criteria. In particular, investment banks are legally confined to communicate to official institutions governing the Italian financial market, the sector each mutual fund belongs

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to as well as a reference index, named *benchmark*. The scope of benchmark is twofold: a) a reference parameter for the portfolio manager when selecting single assets composing its portfolio, b) an indicator of the evaluation of the portfolio management activity for the client, since the returns of each product might not diverge too much from those of the benchmark, especially when negatives.

Aim of this paper is to propose a method for the ranking of Mutual Funds belonging to a specific investment category. In practice, we introduce a criterion able to assess the distances among portfolio composition of different Mutual Funds with respect to the composition of a reference portfolio benchmark, using a constrained rolling linear regression model and the *Mean Integrated Squared Error (MISE)*.

As an example, we consider 39 Italian Mutual Funds operating in the European equities sector, observed weakly from January 4, 1999 to September 30, 2002 (195 observations) together with the Morgan Stanley Europe Index (*benchmark*) and the its constituents. These corresponds to the 10 sectors the whole index is broken into, namely: Energy, Materials, Industrials, Consumer Discretionary, Consumer Staples, Health Care, Financials, IT, Telecommunication Services, Utilities.

2. The Model

We denote with h the number of weeks composing the moving window. We set $h=52$, corresponding to the number of weeks in a year. Moreover, $t=h+1, \dots, T$ indicates time occasions ($T=195$), $f=1, \dots, F$ indicates a generic Mutual Fund ($F=39$) and $s=1, \dots, S$ indicates the generic benchmark sector ($S=10$). As a consequence, $T-h$ denotes the number of times the rolling regression model is applied. With our data, we have $T-h=143$ occasions. Finally, $r_{BMK[h \times 1]}^{(j)}$ denotes the vector of benchmark returns ($j=1, \dots, T-h$), $r_{CONST[h \times S]}^{(j)}$ is the matrix of the benchmarks sector indexes returns and $r_{FUND[h \times 1]}^{(f, j)}$ is the vector of the returns of the generic Mutual Fund f .

Hereinafter, we briefly recall the three steps composing our procedure.

STEP 1: Estimation of the benchmark sector composition. We refer to the following constrained multiple linear regression model:

$$\left\{ \begin{array}{l} r_{BMK[h \times 1]}^{(j)} = r_{CONST[h \times S]}^{(j)} \beta_{BMK[S \times 1]}^{(j)} + \varepsilon \quad j = 1, \dots, T-h \\ \text{with the constraints:} \\ \sum_{s=1}^S \beta_{BMK_s}^{(j)} = 1 \\ 0 \leq \beta_{BMK_s}^{(j)} \leq 1 \quad s = 1, \dots, S \end{array} \right. \quad (1)$$

Each coefficient $\beta_{BMK_s}^{(j)}$ corresponds to the benchmark composition with respect to the sector s . Since the rolling estimation process is repeated $T-h$ times, we obtain $T-h$ vectors of coefficients $\beta_{BMK_s}^{(j)}$. They are juxtaposed in the following matrix (for each j ,

$$0 \leq \hat{\beta}_{BMK_s}^{(j)} \leq 1 \text{ and } \sum_{s=1}^S \hat{\beta}_{BMK_s}^{(j)} = 1):$$

$$\mathbf{B}_{BMK_{[S \times (T-h)]}} = \begin{bmatrix} \hat{\beta}_{BMK_1}^{(1)} & \dots & \hat{\beta}_{BMK_1}^{(j)} & \dots & \hat{\beta}_{BMK_1}^{(T-h)} \\ \vdots & & \vdots & & \vdots \\ \hat{\beta}_{BMK_S}^{(1)} & \dots & \hat{\beta}_{BMK_S}^{(j)} & \dots & \hat{\beta}_{BMK_S}^{(T-h)} \end{bmatrix} \quad (2)$$

STEP 2: Estimation of the Mutual Funds sector composition. Likewise step 1, we estimate the sector composition of each Mutual Fund, using the following constrained multiple linear regression model:

$$\left\{ \begin{array}{l} r_{FUND_{[h \times 1]}}^{(f,j)} = r_{CONST_{[h \times S]}}^{(f,j)} \beta_{FUND_{[S \times 1]}}^{(f,j)} + \varepsilon \\ \text{with the constraints:} \\ \sum_{s=1}^S \beta_{FUND_s}^{(f,j)} \\ 0 \leq \beta_{FUND_s}^{(f,j)} \leq 1 \end{array} \right. \quad \begin{array}{l} f = 1, \dots, F \\ j = 1, \dots, T-h \\ s = 1, \dots, S \end{array} \quad (3)$$

In this case, for the generic Mutual Fund f , we obtain a vector of coefficients expressing its portfolio composition with respect to the generic sector s . Since the rolling estimation process is repeated $j=1, \dots, T-h$ times, the $T-h$ vectors of coefficients $\beta_{FUND_s}^{(f,j)}$ are juxtaposed in the following matrix (for each j , $0 \leq \hat{\beta}_{FUND_s}^{(j)} \leq 1$ and $\sum_{s=1}^S \hat{\beta}_{FUND_s}^{(j)} = 1$):

$$\mathbf{B}_{FUND_{[S \times (T-h)]}} = \begin{bmatrix} \hat{\beta}_{FUND_1}^{(f,1)} & \dots & \hat{\beta}_{FUND_1}^{(f,j)} & \dots & \hat{\beta}_{FUND_1}^{(f,T-h)} \\ \vdots & & \vdots & & \vdots \\ \hat{\beta}_{FUND_S}^{(f,1)} & \dots & \hat{\beta}_{FUND_S}^{(f,j)} & \dots & \hat{\beta}_{FUND_S}^{(f,T-h)} \end{bmatrix} \quad (4)$$

STEP 3: Ranking of Mutual Funds Performance. Both the estimation of the benchmark portfolio composition (step 1) and of each Mutual Fund portfolio composition (step 2) allows to rebuild all the Mutual Funds portfolios and to compare them with the benchmark portfolio composition. This comparison is made using the *Mean Integrated Squared Error (MISE)*, measuring the proximity between an estimated curve $\hat{m}$ and the true function m in the framework of regression smoothing through kernel functions (Härdle; 1990). We denote with f a marginal density and with w a weight function. The MISE, denoted with d_M can be computed as the expectation of the *Asymptotic Squared Error (ASE)*, denoted with d_I , namely:

$$d_M(\hat{m}, m) = E\{d_I(\hat{m}, m)\}; \text{ with } d_I(\hat{m}, m) = \int (\hat{m}(x) - m(x))^2 f(x) w(x) dx \quad (5)$$

We are assuming that the benchmark portfolio time series returns is the true function m to be estimated using the estimated portfolio composition of each Mutual Fund. This assumption allows to discriminate between two different management styles:

1. *passive management*: the investment style of the asset manager is driven by the benchmark portfolio composition, and the corresponding MISE is small;

2. *active management*: the investment style of the asset manager is not necessarily driven by the benchmark portfolio composition, and the corresponding MISE is high.

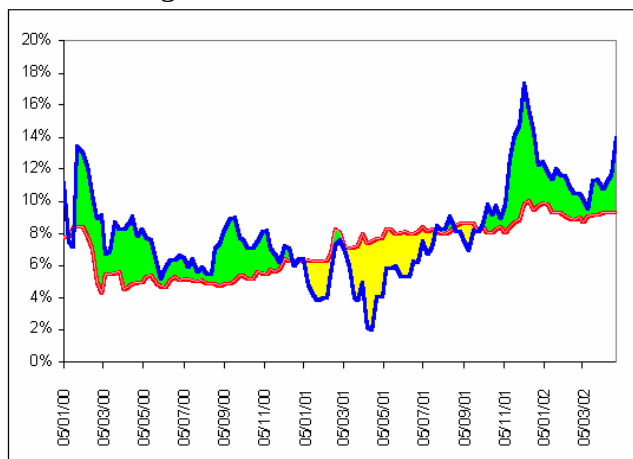
This comparison is done for two main reasons. First, Mutual Funds presenting a small MISE refers to an investment style which is coherent with the benchmark composition (i.e. to the financial market movements), and it should at least provide the same results of benchmark in terms of risk and/or return. Second, the information about portfolio composition can be used in a perspective way. The estimation of the sector composition of both the benchmark and the Mutual Funds portfolios can be used to compose a portfolio of mutual funds (MFMP) on the basis of a pre-specified risk/return target.

3. An application to the Italian Mutual Funds

With regards to the above described data (mutual funds belonging to the European equities sector) the proposed method allows to:

1. estimate the composition of the benchmark with respect to the selected 10 macro sector;
2. estimate the composition of each fund with respect to the same sector composition used for the benchmark;
3. to compare the composition of each fund with respect to the composition of the benchmark by ranking the funds according to the value of the global MISE;
4. to have some information about the details of the management style of the funds, i.e. to examine the way the MISE is determined by positive and by negative extra-returns with respect to the benchmark and with respect to each sector.

The following figure shows the global benchmark trend (red line) with respect to a particular mutual fund (blue line), the green-highlighted area being the positive extra-returns and the yellow-highlighted area being the negative extra-returns, while the sum of such areas being the MISE of the particular fund. Similar measures can be calculated both for the global index and for each reference sector of the benchmark.



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