

On the Holding Period Distribution as a Value at Risk Measure

*Sulla Distribuzione della Durata degli
Investimenti nella Stima del Valore a Rischio*

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Riassunto: Il principale strumento di misurazione dei rischi di mercato relativi al portafoglio di negoziazione delle Banche è rappresentato dal *Value at Risk* (VaR). Sono previsti diversi approcci, ognuno con un proprio grado di sofisticazione ed affidabilità, i quali prevedono alcuni parametri da fissare preventivamente tra cui l'orizzonte temporale dell'investimento, la cui scelta risulta spesso arbitraria e quindi legata dalla tipologia di portafoglio detenuta e dal particolare contesto di mercato. Il modello sviluppato in questo lavoro intende proporre una misura di VaR direttamente legata alla durata dell'investimento, introducendo il concetto di *Zero Risk Line* che definisce il tempo necessario per smobilizzare in utile, o quantomeno non in perdita, le posizioni aperte.

Keywords : Holding Period, Value at Risk, Zero Risk Line

1. Introduction

Value at Risk (VaR) has become in the last few years a standard risk management technique used to measure market risk within financial institutions. Simply defined, VaR is an estimate of the expected maximum potential loss over a given holding period and with a pre-specified confidence level. Many approaches to VaR calculation are available resulting in radically different VaR estimates mostly depending on data, parameters and distributional assumptions.

One of the key variable in VaR calculation is the time horizon to be considered. Capital adequacy standards, especially given by the Basle Committee on Banking Supervision, indicate that the minimum holding period to be used is ten trading days and “Banks may use VaR numbers calculated according to shorter holding periods scaled up to ten days by the square root of time”. Moreover, risk models should consider multiple horizons which are typical of portfolios containing assets with different liquidity profile and useful in treating risk during extreme market moves or distress conditions. This paper is concerned with an alternative approach to VaR measurement directly based on the holding period distributional assumptions.

2. Modelling Value at Risk

To investigate the statistical properties of investment horizons we indicate with the term “Zero Risk Line” a technical line which defines the break even point of an investment or a portfolio, that is the time required to reverse the position with no losses. In particular, let t be a specified date in the sequence of observations ($t \in T$), g_h a new variable that determines the time gap between subsequent data points

$$g_h = h - t \quad t < h \leq T \quad (1)$$

and $r_{h,t}$ the corresponding logarithmic returns of the observed time series P_t at time h

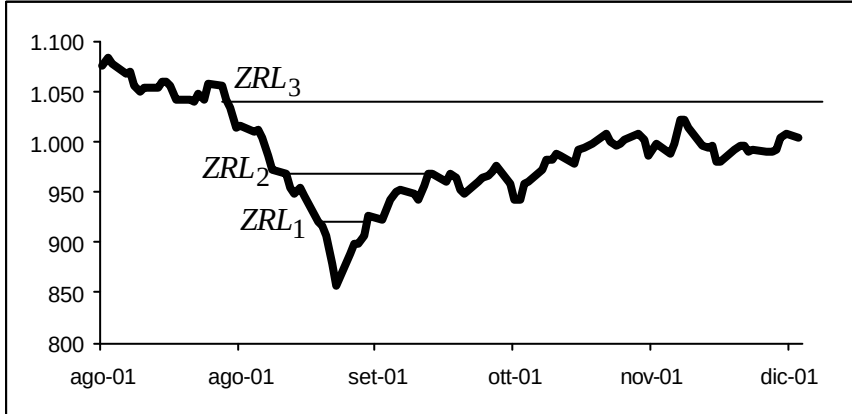
$$r_{h,t} = \ln\left(\frac{P_h}{P_t}\right) \quad t < h \leq T \quad (2)$$

Algebraically, we may represent Zero Risk Lines as follows

$$ZRL_t = \begin{cases} \min[g_h] & r_{h,t} \geq 0 \\ \text{undefined} & r_{h,t} < 0 \end{cases} \quad \forall h \quad t < h \leq T \quad (3)$$

Figure 1 depicts, as an example, three Zero Risk Lines referred to the Morgan Stanley Capital International (MSCI) World Index around the September 2001 market crash: the ZRL_2 line has an higher break even time horizon than the ZRL_1 line whereas the ZRL_3 line describes a position with no break even point at the end of year 2001.

Figure 1: MSCI World Index Zero Risk Lines



The examination of Figure 1 gives evidence of the fact that risk source depends directly on the ZRL distribution ($t \in T - 1$) for different values of h . In order to determine the appropriate holding period it is possible to tabulate the empirical ZRL distribution and then examine the extreme values of that distribution. In particular, one needs to choose a rolling window of observations, typically three to five years of daily data, and then cut off the ZRL distribution at the desired percentile

$$c = \Pr(ZRL \geq \hat{h}) \quad (4)$$

where $\hat{h}$ is the selected holding period and c the specified confidence level. VaR calculation can be performed by tabulating the empirical return distribution referred to the selected holding period and then cutting it off at the desired percentile

$$c = \Pr(r_{\hat{h}} \leq VaR) \quad (5)$$

3. Empirical Evidence

The empirical evaluation is carried out using the returns of a selection of three MSCI stock market indices (one from European Countries (Italy) – one from Emerging Markets (Brazil) and the last one from Usa) along with the World Index. Daily data spans the period from July 1999 to June 2003. All indices are denominated in US dollars. The model is tested monthly over the period January 2001 – June 2003 and aims to quantify the potential impact on risk estimation in comparison to standard RiskMetrics™ methodology.

Table 1 contains the returns summary statistics on all the selected indices (for reason of space only the results at the end of the testing period are reported): according to the Jarque-Bera statistic, we reject the normality hypothesis for all countries (including the World index).

Table 1: Analysis of Country Daily Returns – June 2003

Country	Mean	Standard Deviation	Skewness	Excess Kurtosis	Jarque Bera
Italy	-0,0003	0,0146	-0,1179	1,4414	92,62
Usa	-0,0004	0,0139	0,1488	1,3830	86,80
Brazil	0,0002	0,0159	-0,1225	1,6205	116,28
World Index	-0,0004	0,0106	0,0948	1,3263	77,93

To carry out the VaR calculation we have to consider first the ZRL distribution: Figure 2 describes the ZRL 95% confidence level holding period for all the indices over the whole period whereas Table 2 reports summary statistics for the June 2003 analysis.

Figure 2: ZRL 95% confidence level holding period

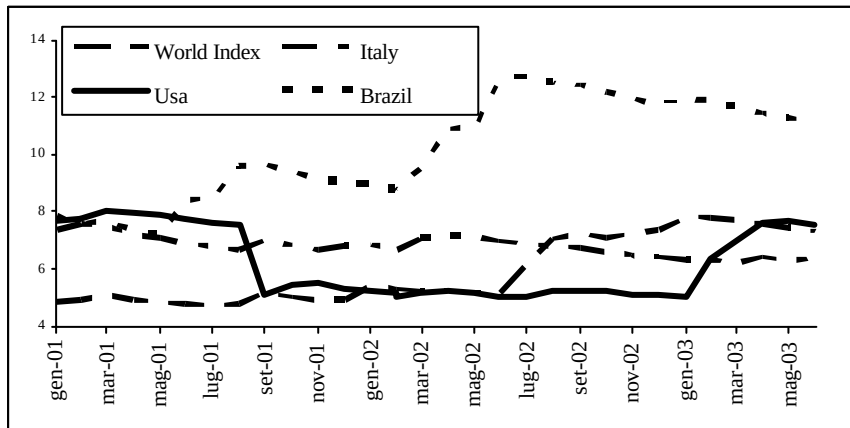
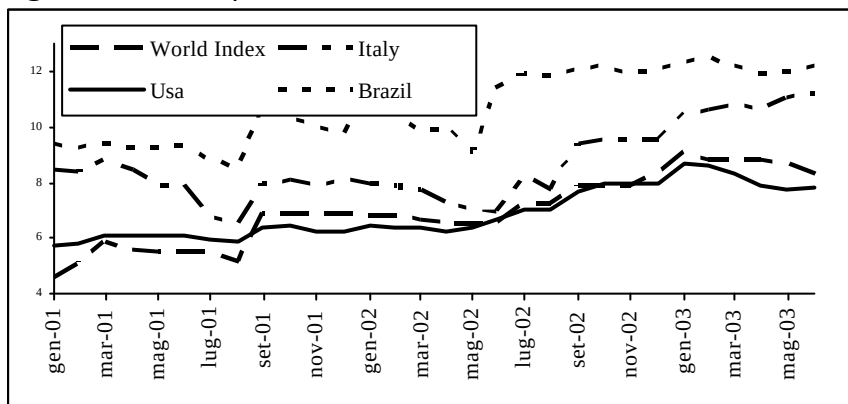


Table 2: Analysis of Country ZRL distribution – June 2003

Country	Mean	Standard Deviation	Skewness	Excess Kurtosis
Italy	6,8053	17,4583	6,7493	44,7917
Usa	7,5089	24,3697	9,1054	82,6837
Brazil	11,1632	35,1733	7,1803	50,0578
World Index	7,3362	21,6158	9,1347	86,9716

As for the VaR calculation, we have to cut off the return distribution associated to the selected holding period: Figure 3 illustrates the 95% VaR for all the indices whereas Table 3 collects the VaR results for the June 2003 analysis compared with the one retrieved from the standard JP Morgan RiskMetrics™ approach.

Figure 3 95% confidence level VaR**Table 3: Comparison of alternative VaR measures – June 2003**

Country	RiskMetrics™ VaR (%)	Zero Risk Line VaR (%)
Italy	6,211	11,213
Usa	5,476	7,865
Brazil	5,398	12,218
World Index	4,522	8,356

ZRL estimates indicate in general higher expected losses than does the RiskMetrics™ technique: over the whole testing period this happens in 73% of simulations. Moreover, backtesting results indicate that ZRL seems to produce more adequate risk views over longer time horizons and when returns distributions deviate from normality.

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