

Hierarchical random effect models for coastal erosion of cliffs

Modelli gerarchici a effetti casuali per l'erosione costale di scogliere

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Riassunto: Il programma di monitoraggio della costa di Holderness, iniziato nel 1951, fornisce un'inestimabile sorgente di informazioni spazio-temporali del fenomeno dell'erosione costiera. Capire e prevedere l'entità dell'attività erosiva è importante per un'adeguata pianificazione dell'uso del territorio. Fino a poco tempo fa i metodi previsivi utilizzati nell'ambito dell'erosione costiera sono stati di tipo deterministico; solo negli ultimi anni si è cominciato ad utilizzare semplici modelli probabilistici per catturare la forte variabilità del processo erosivo. Il nostro scopo, presentando un modello gerarchico a effetti casuali, è quello di migliorare tali modelli cercando di utilizzare al meglio le conoscenze sulle dinamiche dell'erosione costiera. Si presenta anche una soluzione al problema dei dati mancanti, attraverso tecniche di Reversible Jump MCMC.

Keywords: Hierarchical Model, Monte Carlo Markov Chain, Coastal Erosion, Holderness Coast.

1. Introduction

Recession of coastal cliffs presents significant risks to people and property. For example, Sunamura (1992) reports that problems of average cliff recession rates in excess of 1m/year are present in Denmark, Germany, Russia, Japan, New Zealand, Canada, the UK and the USA. The cumulative effects of failures can be dramatic in terms of cliff retreat. For example, the Holderness coast, in the north-east of England, has retreated by around 2 km over the last 1000 years, destroying at least 26 villages.

An awareness of possible cliff position at some future date is fundamental to coastal planning, in particular to avoid locating new development in vulnerable areas (Lee *et al.*, 2001). On the Holderness coast a programme of cliff recession measurement was started in 1951 on an annual basis. 118 erosion posts were installed along the coastline and the cliff top recession was noticed. The resulting dataset provides an invaluable source of information, both spatial and temporal (Pethick, 1996).

Until some time ago, only deterministic approaches were used to provide predictions of cliff recession in terms of sea-level rise, wave-energy, water levels and cliff strength. But they do not reflect the variability of the coastal erosion process as probabilistic models allow. This led to the development of simple stochastic models for the cliff recession process (Hall *et al.*, 2002). Our aim is to improve on these models by taking better account of the known dynamics of the process.

2. Simulation of the coastal recession process

In order to express a formulation of the recession process consistent with known physics it is necessary to understand the basic dynamics of the process. The erosion starts at the base of the cliff: waves break at the foot of the cliff creating a wave-cut notch. As the notch increases in size the weight of the cliff above becomes too much and the cliff collapses.

In other words, an unobservable process of two competitive forces causes failure events: for each cliff loss there must be a cohesive strength that is overcome by the cumulative damage of marine storms. The size of the notch may imply more than one year passes before the next failure occurs: the cumulative damage will then be the sum of the annual damages over this period. The behaviour of these periods of inactivity depends on the geomorphological cliff structure and on the meteorological cumulative damage magnitude.

Let F_w be the observation in year w ($w = 1952, \dots, 2002$) and let $i = i(w)$ be the index of failures ($i = 1, \dots, l$) such that, if in year w a landslide occurred, i would be the progressive number of landslides from $w = 1952$.

Then, let S_i be the i -th failure, let d_{ij} be the annual damages in order to observe S_i ($j = 1, \dots, N_i + 1$) and let N_i be the years of inactivity before S_i . Let R_i be the strength of the rock that must be overcome by the cumulative damage D_i in order to observe the S_i , so:

$$F_w = \begin{cases} 0 & \text{if } \sum_{j=1}^k d_{ij} < R_i \\ S_i & \text{otherwise} \end{cases}$$

where k is the number of years from the previous failure. If S_i occurs, $k = N_i + 1$ and $D_i = \sum_{j=1}^{N_i+1} d_{ij} > R_i$.

Instead of modelling F_w , that is a set of zeros and non-zero values, it is more convenient to model both failure sizes S_i and periods of inactivity N_i between landslides. With regard to model the failure size, looking for example at Site 33 in Figure 1, it seems that after a block of zeros it is more probable to observe a large failure, so it may be a good idea to relate the mean of S_i to the cumulative damage D_i . If we suppose $d_{ij} \sim \text{Exp}(1)$, annual damages can be considered as the points of a unit Poisson process in the interval $[0, R_i]$ and N_i the number of such points.

Consequently, the following hierarchical random effect model can be written down:

1. $S_i \sim \text{Ga}(e^\delta, e^{-(\alpha - \delta + \beta \cdot D_i)})$, $N_i \sim \text{Pois}(R_i)$;
2. $R_i \sim \text{Exp}(\lambda)$, $X_i \sim \text{Exp}(1)$;
3. $\lambda \sim \text{Ga}(a, b)$, $\alpha \sim N(0, d)$, $\beta \sim N(0, e)$, $\delta \sim N(0, f)$;

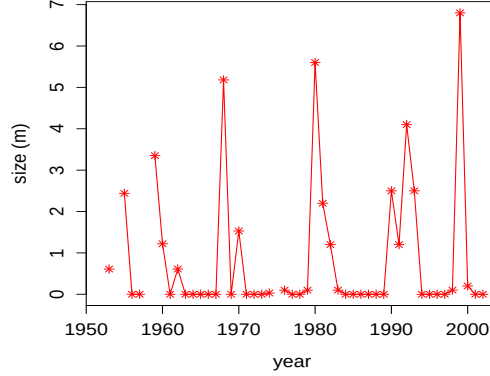
where a, b, d, e, f are constants such that the prior distributions are vague. In the above model λ is related to the tendency of the rock of collapse, α to the magnitude of landslides due to the rock structure and β to the effect of the cumulative damage on the size of landslides after a block of zeros.

MCMC techniques (Gilks *et al.*, 1996) were exploited to make inferences on the model.

3. Parameter estimation in Site 33

As an illustration we show the performance of the model on data from Site 33. This site was chosen because there are few missing data (at the beginning of the time series) and

Figure 1: *Observed failures at Site 33.*



because the data show a typical behaviour.

The estimated expected value of the posterior distribution of α is approximately zero while that of β is approximately 0.25, suggesting that at least in Site 33 the model structure we have proposed in terms of a cumulation of damaging effects is reasonable. The estimated expected value of the posterior distribution of λ is quite small ($\cong 0.51$): indeed, Site 33 is in the North of the Holderness coast where cliffs are comprised of more resistant material, which perhaps also explaining the low estimate of the posterior expected value of α .

Diagnostics suggest a good performance of the estimated model.

4. Missing data

The missing value rate is rather high across sites, averaging 23.5%. In particular, from site 11 to site 31 this rate is always bigger than the mean value. Indeed, from year 1971 to year 1983 measurements have not been collected in most of the first 31 post locations. The problem of missing data leads to a reduction of the already few available observations and splits the time series into short parts. Moreover, if there is a missing value the following value will be the sum of two years' erosion and the parameters set dimension is consequently unknown: Reversible Jump MCMC techniques provide a solution (Green, 1995).

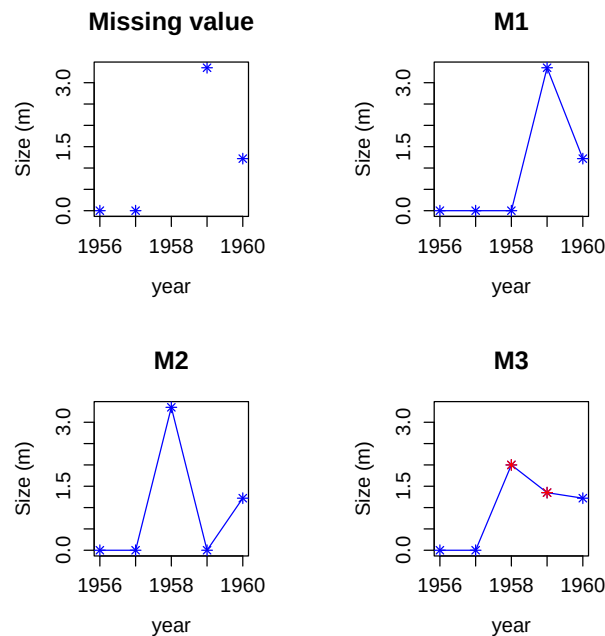
For example, as shown in Figure 2, for Site 33 in years 1956-1960, three configurations are possible. In configuration M3, when compared with M1 or M2, there are another 3 parameters: a value $u \sim U(0, 1)$ that is used to split the sum of the erosion of years 1958 and 1959 (3.35 m) in two failures and another pair of random effects. Otherwise, moving from M1 to M2 (and viceversa), that is a unique landslide of 3.35m is supposed, the parameter set dimension does not change.

Therefore, deterministic changes are necessary in the number of years of inactivity when moving from one configuration to another: in M1 $\mathbf{N} = (3, 0)$, in M2 $\mathbf{N} = (2, 1)$ and in M3 $\mathbf{N} = (2, 0, 0)$.

The extension of the MCMC algorithm to handle missing data enables model infer-

ence at all sites, and as a by-product also provides posterior inferences on the likelihood and magnitude of cliff recession in years for which data are unavailable.

Figure 2: *Three possible configurations for 1958.*



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