

Time series convergence within I(2) models: the case of daily long term bond yields in the euro area

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Abstract: The purpose of the paper¹ is to introduce a modelling strategy that can be used to analyse the process of convergence among time series. The question of time series convergence over an observation period is dealt within cointegration analysis, allowing for an I(2) common trend and a linear broken trend. The modelling strategy is applied to the observed convergence of long-term bond markets within the Economic and Monetary Union (EMU) during the years of the second stage, covering the period from 1 January 1994 to 31 December 1998, before the introduction of euro. The data refer to daily long term bond yields in France, Germany, Italy and Spain over the whole period from 1993 to 2008², that is through the years leading to the monetary union in 1999 and to the monetary unification in 2002, and before the financial crisis of 2008.

Keywords: I(2) common stochastic trends, polynomially cointegrating relations, time series convergence, high frequency financial data

1. The analysis within the I(1) model

The situation of concern is the one where the observed time series show a non-stationary behaviour with common stochastic trends and with different linear deterministic trends over a first sub-period and a non-stationary behaviour, common stochastic trends and no deterministic linear trends over the remaining period. The first approach is to assume that the observed variables $\mathbf{x}_t$ are generated by a Cointegrated Vector Auto-Regression (CVAR) model with broken linear trends both in the data and in the cointegrating relations (Juselius, 2006, p.297):

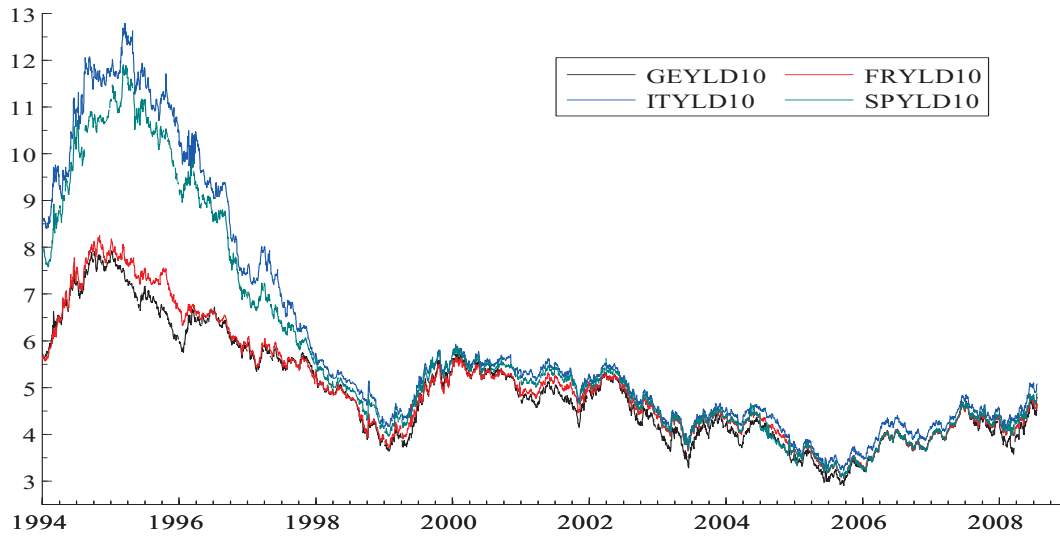
$$(1) \quad \Delta \mathbf{x}_t = \Gamma_1 \Delta \mathbf{x}_{t-1} + \alpha \tilde{\beta}' \tilde{\mathbf{x}}_{t-1} + \mu_0 + \varepsilon_t, \quad \varepsilon_t \sim N_p(\mathbf{0}, \Omega), \quad t = 1, 2, \dots, T,$$

where $\tilde{\beta}' = [\beta', \beta_{11}, \beta_{12}]$ $\tilde{\mathbf{x}}'_t = [\mathbf{x}'_t, t, tD_{t^*}]$. When applying this approach to our data set, represented in Figure 1, after having filtered the observations using a centred simple moving average calculated over three consecutive observations and taking as t^* the first observation of 1999³, the cointegration analysis results give as reasonable the existence

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Figure 1: Ten-year zero-coupon bond yields for Germany, France, Italy and Spain



Source: Ehrmann et al. (2007)

of two cointegrating relations and two common stochastic trends. Trying to identify the long-run structure, that is restricting the cointegration space in order to get uniquely identified stationary relations, the resulting normalised long-run relations, slightly accepted with a p -value = 0.029, are given by:

$$(2) \quad \hat{\beta}'_1 \tilde{x}_t = 0.820(FRYLD10_t - GEYLD10_t) - 0.180(ITYLD10_t - FRYLD10_t) + 0.00034t_{04:01:1999} - 0.00034t$$

$$(3) \quad \hat{\beta}'_2 \tilde{x}_t = 0.806(FRYLD10_t - GEYLD10_t) - 0.194(SPYLD10_t - FRYLD10_t) + 0.00025t_{04:01:1999} - 0.00026t$$

showing that, when corrected for the difference in the slope coefficients of the deterministic trends, weighted differences between spreads seem to be stationary, instead of the single spreads, as we would expect. These results are a clear indication that we should consider the curvature of the yield spreads and not just the slope of them, as in Giese (2008), where the author found similar results analysing monthly data of US treasury zero-coupon bonds yields of five different maturities.

2. The analysis within the I(2) model

The methodological approach followed, that is trend-adjusting for the change in regime and treating the series as I(1), has proven unsatisfactory in terms of both identification and stationarity of the cointegrating relations. In particular, their graphs show some non-stationarity over the period of observation. These, together with the graphs of the

⁽³⁾ Given the high frequency of data, there have been some problems performing formal misspecification tests for assessing the adequacy of the VAR specification, particularly when checking the multivariate normality assumption and the residual autocorrelation.

data in first and second differences and the characteristic roots of the model, are clear signals that we should consider also the possibility of a double unit roots in the time series, that is analysing them as I(2) variables⁴. As Juselius (2006, p. 293) explains "... the typical smooth behaviour of a stochastic I(2) trend can often be approximated with an I(1) stochastic trend around a broken linear deterministic trend ...". Therefore, the second approach is to model the regime change stochastically within a cointegrated I(2) model, as follows (Juselius, 2006, p.319):

$$(4) \quad \Delta^2 \mathbf{x}_t = \alpha(\tilde{\beta}' \tilde{\mathbf{x}}_{t-1} + \tilde{\delta}' \Delta \tilde{\mathbf{x}}_{t-1}) + \zeta \tilde{\tau}' \Delta \tilde{\mathbf{x}}_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N_p(\mathbf{0}, \Omega),$$

where, on the basis of the I(2) rank test statistics, it was decided for $r=3$ polynomially cointegrating relations and for $(p-r)=1$ common I(2) stochastic trend, and no I(1) stochastic trend. Quite surprisingly, the estimation iterative procedure has reached the final estimates in few iterations, while for any other choice for r and $(p-r)$, the number of iterations taken is really very high. The identified dynamic long-run equilibrium relations are the following:

$$(5) \quad \begin{aligned} \hat{\beta}'_1 \tilde{\mathbf{x}}_t + \hat{\delta}'_1 \Delta \tilde{\mathbf{x}}_t = & FRYLD10_t - GEYLD10_t - 0.0005t_{02:01:1999} + 0.0005t + \\ & 9.634\Delta GEYLD10 + 9.634\Delta FRYLD10 + 9.636\Delta ITYLD10 + \\ & 9.636\Delta SPYLD10 - 0.423\Delta t_{04:01:1999} - 0.796\Delta t \end{aligned}$$

$$\begin{aligned} \hat{\beta}'_2 \tilde{\mathbf{x}}_t + \hat{\delta}'_2 \Delta \tilde{\mathbf{x}}_t = & ITYLD10_t - GEYLD10_t - 0.0043t_{02:01:1999} + 0.0043t + \\ & 42.057\Delta GEYLD10 + 42.061\Delta FRYLD10 + 42.092\Delta ITYLD10 + \\ & 42.086\Delta SPYLD10 - 1.299\Delta t_{04:01:1999} - 9.267\Delta t \end{aligned}$$

$$(6) \quad \begin{aligned} \hat{\beta}'_3 \tilde{\mathbf{x}}_t + \hat{\delta}'_3 \Delta \tilde{\mathbf{x}}_t = & SPYLD10_t - GEYLD10_t - 0.0036t_{02:01:1999} + 0.0036t + \\ & 38.711\Delta GEYLD10 + 38.714\Delta FRYLD10 + 38.738\Delta ITYLD10 + \\ & 38.734\Delta SPYLD10 - 1.301\Delta t_{04:01:1999} - 7.570\Delta t \end{aligned}$$

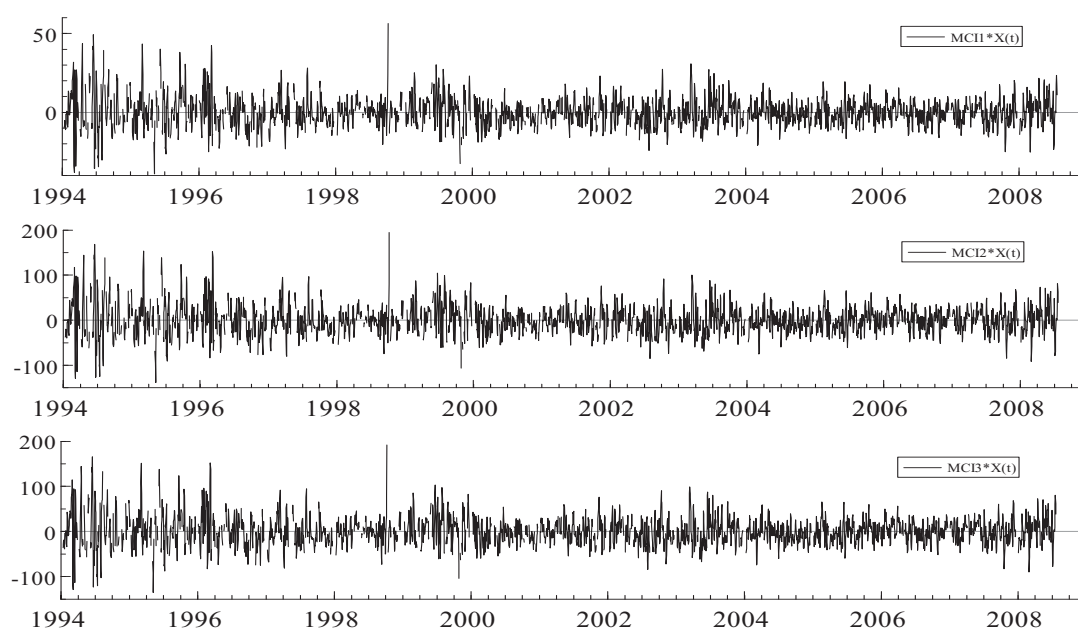
The LR test statistic on the over-identifying restrictions has given a value $\chi^2_4 = 2.534$, with a p -value = 0.639, making the restrictions largely accepted. The restricted polynomially cointegrating relations are reported in Figure 2, that shows their stationarity over the sample. The identified relations highlight the fact that the spreads between Germany and each other country considered are stationary as expected, when allowing for a broken linear trend, short-run adjustments and an I(2) component.

Testing the restriction whether a linear trend is needed in the sub-period going from the beginning of 1994 to the end of 1998, but not in the following sub-period, that is just a broken linear trend, the results show that for each polynomially cointegrating relation the hypothesis is largely accepted. This can be interpreted as a clear signal that the convergence in the long-term bond yields has been achieved by January 1999. After that date, the data show no significant linear trends and the eventual deterministic components imply only that the equilibrium means are different from zero.

When analysing the adjustment dynamics with respect to the polynomially cointegrating

Figure 2: *The three polynomially cointegrating relations*

⁽⁴⁾ A formal treatment of I(2) models and relative tests can be found in Johansen (1997).



relations, it is interesting to note that first differences in French, Italian and Spanish bond yields, are significantly equilibrium correcting to the levels and that second differences in Italian and Spanish bond yields are significantly equilibrium correcting to first differences. Moreover, the estimates show that it is primarily the twice cumulated shocks to the German long-term bond yields which have generated the I(2) stochastic trend.

Conclusions

The bond market unification analyzed implies a single latent factor, that is a single common stochastic trend as a driving force underlying yields of the same maturities across different countries' market. Analyzing the daily observations of long term bond yields in France, Germany, Italy and Spain within the I(2) model we have found evidence of such common stochastic trend given by the twice cumulated shocks to the German long-term bond yields. Such result shows the importance of taking into account the curvature of the bond spreads in the process of convergence and not just the level and the slope. As a conclusion it's reasonable to state that the chosen I(2) approach has allowed a better modelling of the convergence process than within the I(1) model, giving evidence to expected characteristics that the I(1) model wouldn't show.

References

- Ehrmann M., Fratzscher M., Gürkaynak R., Swanson E. (2007) *Convergence and Anchoring of Yield Curves in the Euro Area*, FRBSF Working Paper 2007-24.
- Giese J.V. (2008) "Level, Slope, Curvature: Characterising the Yield Curve in a Cointegrated VAR Model", *E-Economics*, Vol.2, 2008-28.
- Johansen S. (1997) *Likelihood Based Inference in Cointegrated Vector Auto-Regressive Models*, Oxford University Press, Oxford.
- Juselius K. (2006) *The Cointegrated VAR Model. Methodology and Applications*, Oxford University Press, Oxford.