

Mover Stayer Model in a small Industrial Area: a first application ¹

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Abstract: In the last decade the District of Prato has suffered a deep shrinkage of exports and Added Value of its textile industry which is the core of the overall economy of the same small area. (On the basis of Coeweb- ISTAT data base for the years 2001-2009 the annual average rate of total exports is -5,2%). In this paper we want to investigate if the textile crisis has been turning into a firm downsizing -measured in number of employees- of the same industry and if the downsizing has been spreading through the overall economy of the District. For this purpose we use the Mover Stayer Model (MS) whose advantage is the possibility to consider firms heterogeneity, necessarily ignored in classical Discrete Markov Chains. The used estimators are Goodman (1961) and Frydman (1984). Of this second estimator we point out the necessity to restrain its validity both on applied and a priori mathematical ground. Data are represented by two fix panels formed on ASIA-ISTAT. Firm size equilibrium distributions are simulated.

Keywords: Mover-Stayer Model, Fix Panel

1. The Model

Let's start from the following definition: a *Latent Class Model* is a statistical model that relates a discrete random variable X (so-called manifest variable) to a set of latent classes $\{a_j\}$. Then it holds

$$P(X = x) = \sum_j P(x|a_j)P(a_j)$$

The Mover-Stayer Model (MS) is a special case of latent class model: $X(t) \in \{1, \dots, I\}$ is the value at time t of a given variable associated to every individual in a given population. The individuals are subdivided in two latent classes: Movers (X evolves according to a Markov Chain with transition matrix M) and Stayers ($X(t) \equiv X(0)$). Let s_i the probability of an individual starting from the i -th state to be a Stayer, and let S be the diagonal matrix with diagonal elements s_i , then the global one-step transition matrix is given by the mixture

$$P = S + (I - S)M \quad (1)$$

Calling $P^{(t)}$ the transition matrix at the t -th step, it satisfies no longer the Markov property $P^{(t)} = P^t$, but it satisfies

$$P^{(t)} = S + (I - S)M^t \quad (2)$$

¹We are grateful to ASEL-Provincia di Prato for the permission to use ASIA.

2. Estimation

According to MS, the following couple has to be estimated:

$$(s, M) \in [0, 1]^I \times [0, 1]^{I^2}$$

where s is the vector $(s_1, \dots, s_I)$ and M has been defined above.

We have at our disposal observations about T successive years. Differently from the classical Markov Chain Model, the estimation cannot be direct because an individual which is observed to remain in its starting class might be a Stayer, but also a Mover which has not moved in the span of observed time, and this last event occurs with non-null probability. Seminal work aiming to solve this question has been supplied in Blumen *et al.* (1955), which estimator has been proved to be not consistent in Goodman (1961). Further research has made available new consistent estimators (Goodman (1961), Frydman (1984)). Among these, through applied research, two have been revealed particularly apt to use most of the available information contained in the archives ASIA/ISTAT and CERVED Business Account, and we shall give a brief description of them in the following paragraphs.

Goodman (1961) The proposed estimator for the matrix M has elements:

$$\bar{m}_{ij} = \begin{cases} \left[\sum_{t=1}^T w_{ii}(t) - Tc_i \right] / \left[\sum_{t=1}^T w_i(t) - Tc_i \right] & \text{if } i = j \\ \left[\sum_{t=1}^T w_{ij}(t) \right] / \left[\sum_{t=1}^T w_i(t) - Tc_i \right] & \text{if } i \neq j \end{cases} \quad (3)$$

where, for $t = 1, \dots, T$, $w_{ij}(t)$ is the number of observed transitions from i to j at the t -th step, $w_i(t)$ is the number of individuals in state i at time t and c_i is the number of individuals which continuously remain in the starting state i . The author claims that $\bar{m}_{ij}$ is the MLE of m_{ij} based upon all the data. Once computed $\bar{M}$ it is trivial to estimate s .

Frydman (1984) The estimator of m_{ii} is found minimizing a likelihood function, that is by solving:

$$g(x) = [n_i^* - Tn_i(0)]x^{T+1} + [Tn_i(0) - n_{ii}]x^T + [Tn_i - n_i^*]x + n_{ii} - Tn_i = 0 \quad (4)$$

where $n_i(0)$ is the initial number of individuals in i , $n_i(t)$ is the number of individuals in state i at time t , $n_{ij}(t)$ is the number of individuals in state j at time t which were in state i at time $t - 1$, n_i is the number of individuals which continuously remain in i , $n_{ij} = \sum_{t=1}^T n_{ij}(t)$ and $n_i^* = \sum_{t=0}^{T-1} n_i(t)$. Estimation of m_{ij} and s are recursively founded. We point out that the validity of this estimator has to be narrowed both on mathematical and applied grounds. In fact, the equation 4 has a unique solution within $(0, 1)$ if it holds

$$g'(1^-) = T[n_i^* - n_i(0) - n_{ii} + n_i] > 0. \quad (5)$$

However, as we will see in section 3., it could happen $g'(1^-) = 0$; in this case the solution of 4 is $\hat{m}_{ii} = 1$ and the i -th state is considered as an absorbing one even if it is not true.

3. Empirical Applications

In this section we present two applications of above procedure to ASIA-Textile industry (3465 firms) and ASIA-overall economy (19559 firms). Both the panels are available since 2000 to 2006. We have subdivided the population of firms in eight states, according to the number of employees. (Use of classical Discrete Markov Chains and MS to produce simulations of Wage, Income, Firm Size distributions is in Solow (1951) and Quah (1993)). Because we are space constrained we present only small part of our results: we show tables 1 and 2, containing the χ^2 -Goodness-of-Fit-Test applied to the two estimators, year by year.

Table 1: χ^2_3 -Test Results for ASIA-Textile industry, 95%-critical value=14.07

	'01	'02	'03	'04	'05	'06
G	6.98	12.66	7.41	4.17	0.69	5.38
F	7.77	15.32	11.53	10.15	3.04	2.18

Table 2: χ^2_7 -Test Results for ASIA-Overall economy, 95%-critical value=14.07

	'01	'02	'03	'04	'05	'06
G	4.97	13.63	16.39	8.40	3.42	2.99
F	5.44	21.93	37.41	27.28	25.18	17.66

G and F stand for Goodman and Frydman. We would remark that the Frydman estimation, being recursive, is affected by the propagation of approximation errors and we note that this model, when applied to ASIA-overall economy, is substantially rejected. According to this result we choose to use the G-estimator to obtain the equilibrium distributions. The rows EQ in both tables 3 and 4 display the results respectively for the textile industry and for the overall economy. The "equilibrium distribution equal to the effective distribution" (year 2006) represents an hypothesis which is rejected (95% level) for the textile industry and which is not refused for the overall economy. Our research points out that downsizing affects sharply the textile industry but it does not involve the overall economy. The same results look as a confirm of ISTAT's estimates according to whom the quota of Services in the Added Value of the Province of Prato has been rising in the last decade.

Table 3: Observed 2006-distribution versus Equilibrium in ASIA-textile industry

	(0, 1]	(1, 2]	(2, 5]	(5, 10]	(10, 20]	(20, 50]	(50, 100]	> 100	χ^2
2006	0.218	0.211	0.214	0.167	0.122	0.053	0.013	0.003	38.64
EQ	0.248	0.241	0.219	0.141	0.095	0.044	0.009	0.002	

Table 4: *Observed 2006-distribution versus Equilibrium in ASIA-Overall economy*

	(0, 1]	(1, 2]	(2, 5]	(5, 10]	(10, 20]	(20, 50]	(50, 100]	> 100	χ^2
2006	0.410	0.207	0.205	0.094	0.055	0.022	0.005	0.002	9.513
EQ	0.416	0.206	0.207	0.097	0.049	0.018	0.004	0.002	

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