

Harmonic growth: a new approach to the measurement of countries development*

Daria Mendola, Raffaele Scuderi

Università degli Studi di Palermo

Dipartimento di Metodi Quantitativi per le Scienze Umane

Viale delle Scienze, ed.13- 90128 Palermo, Italy

mendola@unipa.it

scuderi@unipa.it

Abstract: The paper defines *harmonic growth* as a situation of diffuse well-being associated to an increase of GDP, and proposes a measures of time series similarity between GDP and each one from a set of social and economic indicators of development. Such indexes are then synthesized in order to provide the global degree of *harmony in growth* inside a country. A first Harmonic Growth Index is based on comparisons between couples of polynomials fitting the series. The second version of the index is obtained from cubic B-splines estimates. With an accurate selection of social indicators, both indexes can also be used to rank countries according to their level of development, offering effective complementary information to the Human Development Index.

Keywords: growth, time series pattern recognition, indicator, development

1. A Definition of Harmonic Growth

Analysis of Gross Domestic Product has traditionally been a point of reference for studies about growth of Countries. Although this indicator has been widely criticized, scholars still employ it as the main measure of well-being. It is well known and deeply debated in literature that GDP takes no account of aspects such as increasing inequality, pollution, damage to people's health; moreover, many scholars remark how, in a perverse manner, crime and other elements of social breakdown concur in the computation of this richness measure. The need to find an alternative index to GDP is revealed by the success of a measure like Human Development Index, which nowadays competes with GDP as headline indicator of development in world wide ranking (UNDP, 2007).

In this paper we intend growth as synonymous of progress and well-being of a nation, and our idea of “harmony in growth patterns” is inspired by a relatively recent literature about living standards analysis (e.g. Hobijn and Franses, 2001), which considers growth as a multidimensional phenomenon. These studies conceive growth as something related to different social and economic aspects, thus making this concept “wider” than

* Authors wish to thank dr. Valerio Lacagnina who provided precious suggestions for the improvement of the index here proposed. The authors are solely responsible for any error and inconsistency in this paper.

the one related to GDP. As a consequence, measurement of growth is made through a multivariate approach.

Also *harmonic growth* is conceived as a multidimensional concept. Growth of a country happens *harmonically* when a positive growth rate of GDP is associated with fair and positive changes in social indicators (lagged, if it is the case), which globally indicate the spread of wealth and its beneficial effects on population. *Harmonic growth* also happens when a negative growth rate of GDP is associated with negative changes in well being indicators, which point out an overall concentration of wealth and a decrease of social and economic levels of life inside the country. The concept implies that evaluation of *harmony of growth* deals with comparison of time series patterns of several indicators. *Harmonic growth* is different from positive correlation among contemporaneous observations, since it is based on the comparison of the time behavior of series along the whole period of observation. From an operational perspective, the evaluation of harmonic growth requires that comparisons have to be made between GDP and a set of social indicators that are supposed to be complementary measures of well-being (for example in developing countries one could select female employment rate, enrolment rate in primary school, political rights extension, the inverse of infant mortality rate; more in general, the choice of indicators has to be made between those which are directly/positively linked with human development).

Some contributions on pattern similarity literature employ measures such as Euclidean distance or linear correlation between contemporaneous observations, which however are not suitable for time series since they are invariant toward changes in time sequencing of data. A different approach is provided by such techniques based on stochastic processes (such as cointegration and AR or ARIMA metrics – Piccolo, 1990), which instead explicitly puts in account time structure of data. Moreover, an interesting point of view is the one offered by curve clustering approach via B-splines (Abraham *et al.*, 2003). Authors propose a robust method which clusters time series using a reduced number of coefficients estimated through cubic B-splines. As final result, their method provides a set of clusters of time series with similar patterns, which are obtained from a *k-means* algorithm. In this paper we also employ cubic B-splines, together with polynomials, but our aim is to obtain an index which quantifies the degree of similarity (“harmonicity”) between series.

2. A Measure of Harmonic Growth

Let X_t and Y_t be two time series, both with n terms. For instance, let X_t be GDP series and Y_t the female employment rate series. Suppose that X_t and Y_t can be described by polynomials of order $k \leq n-1$ in t , and β and δ are the vectors of the k coefficients of the polynomials related respectively to X_t and Y_t (excluding the constant terms). If two series present a “harmonic” pattern, then $\beta_i - \delta_i$ equals 0 for each i ($i=1, 2, \dots, k$), this indicates that the series are perfectly overlapped or proportional.

The proposed Harmonic Growth Index is:

$$HGI = 1 - \sum_{i=1}^k \frac{|\beta_i - \delta_i|}{2 \max(|\beta_i|, |\delta_i|)} w_i \quad (1)$$

where the absolute differences $|\beta_i - \delta_i|$ are compared to their theoretical maximum, corresponding to perfect disharmony in which series are “opposite” (see panel *a* in Fig.1), and w_i are weights spanning in $[0, 1]$. If we put $w_i = 2i/(k(k+1))$ in (1), weights increase as the order of the monomial to which coefficients β_i and δ_i belong raises. Therefore, the higher the order of the monomial, the higher the importance given to a possible divergence in the compared coefficients, and thus contribution to global disharmony in the common pattern of two series is higher¹.

An alternative version of the index can be computed from B-splines coefficients vectors (HGI_s in the following), where β and δ are the vectors of the h coefficients for the cubic spline basis, and $w_i=1/h$. Note that the resulting index is more robust to outliers or small changes in data in one part of the time domain than HGI computed on polynomials (HGI_p in the following). Coefficients from polynomial regression are instead more sensitive to small changes in data: this could be a good property if one can assume that data are not affected by measurement errors or if small changes are to be highlighted by the index.

3. Properties of the Index and Final Remarks

HGI is a symmetric measure with respect to the role played by the two series compared, consistently with the common sense the idea of *harmony* gives. Both HGI_p and HGI_s range in $[0,1]$, where 0 corresponds to the absence of harmony in dynamics (that is perfect disharmony in time series patterns) and 1 results from perfectly harmonic patterns.

Figure 1 displays four pairs of simulated time series and the corresponding values of indexes². As mentioned above, panel *a* refers to perfectly mirrored series and both HGI equal zero; in fact, according to our definition of *harmonic growth*, this situation corresponds to maximum disharmony: an increase of GDP corresponds to a decrease of the social indicator of well-being. Indexes equal 0.5 when one of the two time series does not vary along the time (Figure 1, panel *b*), which means that GDP dynamics does not affect well-being indicator’s behavior. When series have an identical growth path (raising or decreasing together), indexes equal 1 (see Figure 1, panel *d*).

The more the two series tend to show similar behaviors, the more the indexes tend to their maximum (see Figure 1, panel *c*). At this regard, as said above, the panel *c* shows how HGI_p is more sensitive to small differences in patterns of the two series, and therefore shows smaller value of the index, while HGI_s focuses on their similar trends.

Indexes of harmonic growth upon all the couples of time series (i.e. between GDP and each of the m social indicators selected for a study) can be synthesized into a Global Harmonic Growth Index – GHGI – according to the “weaker ring criterion”:

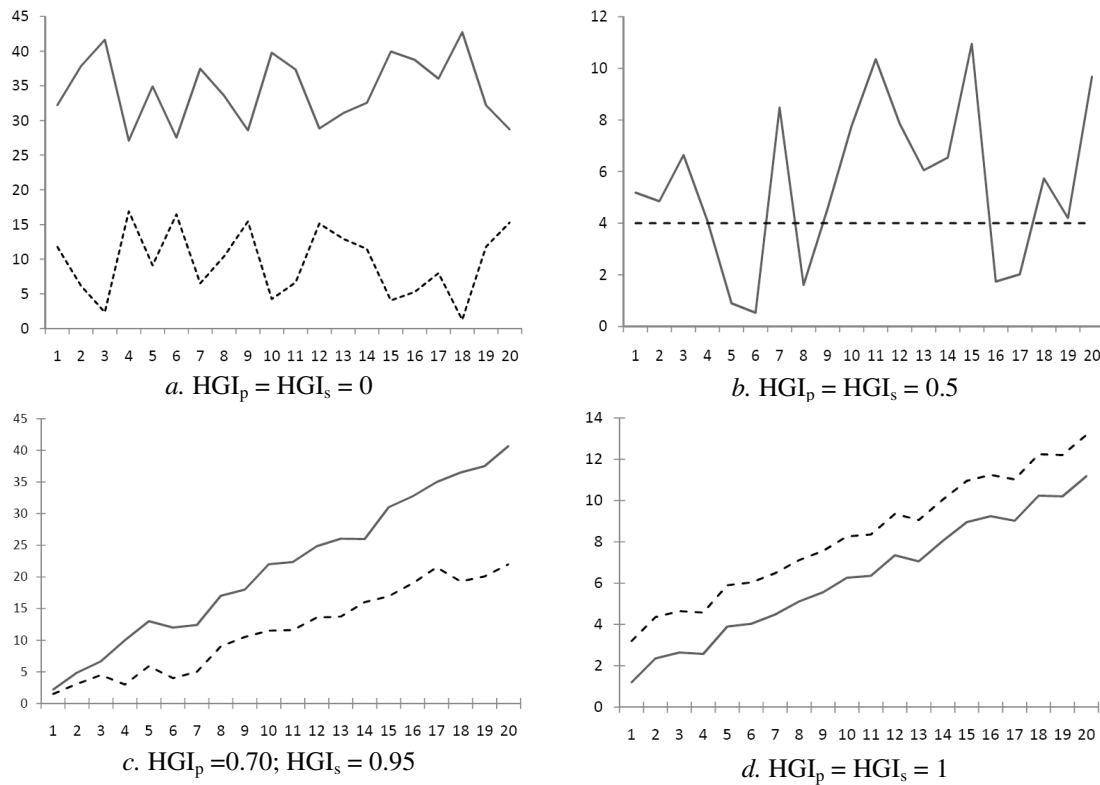
¹ Note that raw OLS estimates of β and δ (**b** and **d**) depend on average magnitude of the series, on their observed range, and on their units of measurement. So before comparing homologous coefficients in the polynomials it is necessary to find a transformation allowing for proper comparisons. According to classical regression theory we propose to adopt standardized coefficients; they provide adimensional estimates and adjust for different units of measurement and different variability and magnitude.

² As know from literature a polynomial of order 6 or 7 usually shows a good fitting to data, capturing all relevant information. This is why indexes referring to raw data reported in Figure 1 are based on estimates of polynomials of degree six.

$$GHGI = \min (HGI_1, HGI_2, \dots, HGI_m) \quad (2)$$

where HGI_i is the index of harmony between GDP and the i^{th} well-being indicator. Alternative aggregation functions could be the mean between HGI_i if one can suppose that a disharmony between a couple of series could be compensated by the higher harmony of another one. Otherwise one could also consider to take into account the median of the m indexes if there are outliers within the set of HGI_i . GHGI provides a measure of the total degree of multidimensional harmony in growth inside a country. GHGI can be used also to rank countries according to their level of development, and it offers complementary information to the Human Development Index, as it is explained in a forthcoming paper.

Figure 1: *Harmonic growth indexes*



References

- Abraham C., Cornillon P.A., Matzner-Lober E., Molinari N (2009), Unsupervised Curve Clustering using B-Splines, *Scandinavian Journal of Statistics*, 30, 581-595.
- Hobijn B., Franses P. (2001) Are living standards converging?, *Structural Change and Economic Dynamics*, 12, 171-200.
- Piccolo D. (1990) A distance measure for classifying ARIMA models, *Journal of Time Series Analysis*, 11(2), 153-163.
- UNDP (2007) *Human development report 2007/2008*, Palgrave Macmillan, New York, <http://hdr.undp.org>