

Groundwater resource monitoring via a Transfer Function approach¹

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Abstract: groundwater monitoring is an important environmental issue for local public administrations. This paper illustrates a proposal for groundwater monitoring elaborated jointly with officers from Regione Piemonte. Daily groundwater levels are strongly affected by exogenous predictors, such as rain precipitation, neighboring waterways or local irrigation ditches. We discuss a transfer function approach applied to monthly series of piezometric levels to model these neighboring effects. The aim of the study is to give Regione Piemonte a criterion to handle groundwater monitoring, using an approach similar to on-line quality control techniques, but in a public context.

Keywords: piezometric levels, missing data, environmental statistics.

1. Introduction

This work is a description of an ongoing project on the statistical methods for groundwater monitoring. It is a joint effort with the environmental department of Regione Piemonte, which is then both a participant and the final user of the results.

Groundwater monitoring is an important issue for upcoming environmental legislation by local public administrations. Regulations may imply, for instance, limitations on private ground watering. In order to make correct decisions, the contribution of natural predictors should be identified first. Groundwatering is a complex phenomenon in which different exogenous predictors, such as the presence of other water sources (neighboring waterways or local irrigation ditches) and rain precipitation, are involved. Due to the geography of the areas and the monitoring stations considered in this project, the level of neighboring rivers is taken as the most important predictor of water-table levels.

More precisely, all time series considered have first been preprocessed in order to eliminate trend and seasonality, and missing data have been reconstructed. Model identification for groundwater levels has then been improved by fitting a transfer function model based on the observation of neighboring rivers, in order to isolate and quantify their immediate

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Monitoring station	River	Observation Time
Torino (Piazza D'Armi)	Dora	Mar, 2005 - Mar, 2008
MonteCastello	Tanaro	Nov, 2000 - Apr, 2008
Carignano	Po	Jan, 2001 - Mar, 2008
Masio	Tanaro	Jul, 2000 - Dec, 2007

Table 1: *groundwaters and next waterways*

influence. Two different models, with and without the effect of neighboring rivers, are then estimated.

Regione Piemonte has shown particular interest in this analysis. The predicted levels, in fact, will represent useful information in order to be able to recognize critical situations among the monitoring stations. The final goal is the determination of critical thresholds for the predicted values, an approach similar to on-line quality control techniques in manufacturing.

2. Data pre-processing

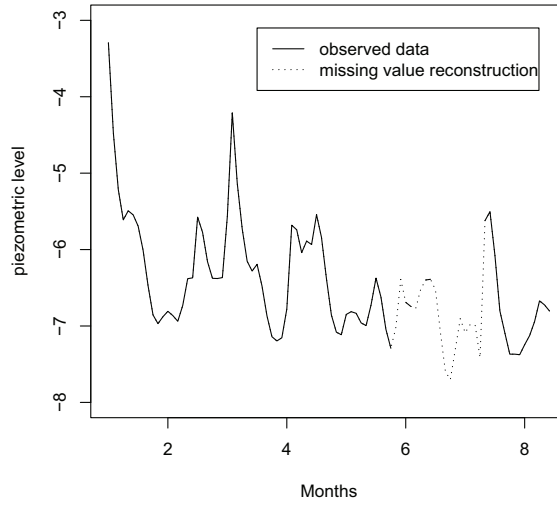
Since 2004, Regione Piemonte has been rapidly increasing the number of piezometers throughout its area, in order to capture information about groundwater resources. Piezometers are special artificial wells designed to measure the underground height of the water-table.

In the analysis presented in this work, special interest is devoted to those piezometers which can be found near waterways and can therefore be immediately affected by them. Table 1 contains the location of the piezometers analyzed and the next waterway considered. Such waterway has actually turned out to be a river in all cases we consider. Piezometric measurements are recorded irregularly, that is, the number of measurements available per day has been changing through the years. Moreover, piezometers are really fragile tools, so that measurements may sometimes be not available for several days. These preliminary problems, together with the final aim to study long period trends, have lead us to consider monthly time series of groundwater levels derived from the original series by averaging the data observed in a given month. Neighboring river level time series have been processed in the same way.

Several statistical issues arise, since the series appear to be highly non stationary and still present missing values. For example, the Monte Castello groundwater time series has still shown a long period of missing values even after the pre-processing steps described above. Because of the high correlation between its groundwater times series and the Tanaro level time series, a suitable multiple regression model has been estimated in which both time series have been involved with different time-lags. The reconstruction obtained can be found in Figure 1.

In order to achieve stationarity, both the seasonal and trend component have been removed from all time series by applying a moving average and LOESS estimation strategy. The remainders have then been used to implement a transfer function model, as described by the next section.

Figure 1: Pre-processing of Montecastello piezometric levels **Montecastello**



3. Modelling the effects of neighboring waterways via transfer function models

Let (X_t) be the monthly time series of groundwater piezometric levels and let (Y_t) be the time series of the corresponding next waterway levels, after the pre-processing steps described in the previous section. (X_t) and (Y_t) can be taken as regular stationary time series. In a transfer function model, a linear relationship is assumed between the two processes through a linear filter, that is

$$Y_t = \sum_{s=0}^{\infty} \nu_s X_{t-s} + N_t := H(B)X_t + N_t \quad (1)$$

where $H(B) = \omega(B)/\delta(B)$ is the ratio of two finite polynomials in the lag operator B and (N_t) is the error process (not necessarily a white noise) representing the part of Y_t which cannot be explained by a linear relationship with (X_t) . The two processes (N_t) and (X_t) are taken as uncorrelated to each other. Notice that the presence of $\delta(B)$ is equivalent to using (Y_t) lags. Identification and estimation for model (1) is described, for example, in Battaglia (2007).

As an example, let us consider here the results concerning the Carignano station. Written in an equivalent way, the final model 1 estimated for this case is

$$Y_t = -1.17Y_{t-1} + 0.51X_t + (1 + 0.76B)e_t, \quad (2)$$

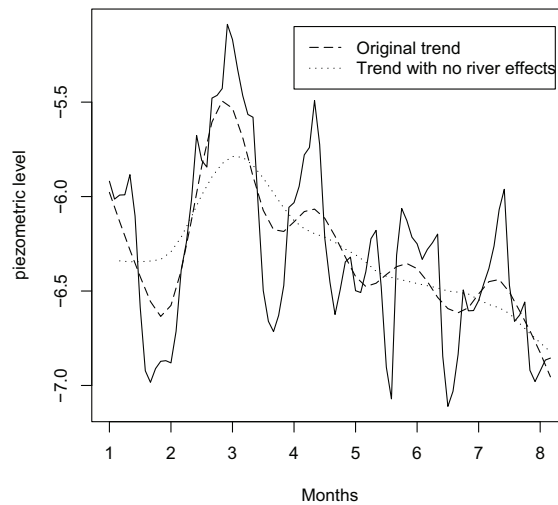
where (X_t) denotes the times series of Po levels next to Carignano and (e_t) is a white noise. In particular, notice that in our fitting we identify $(N_t) = (1 + 0.76B)e_t$ with a moving average of an independent white noise process.

In order to remove the river effect on groundwater levels, the following proxy process $\tilde{Y}_t$ is then considered:

$$\tilde{Y}_t = -1.17\tilde{Y}_{t-1} + (1 + 0.76B)\hat{e}_t, \quad (3)$$

where $\hat{e}_t$ are the fitted residuals for the model 2. Finally, the proxy $\tilde{Y}_t$ is added back to the trend and seasonal components estimated in the pre-processing step in order to get a

Figure 2: *Comparison between old trend and “virgin” trend.* Trend Carignano



groundwater “virgin” level, in which the Po immediate effects have been removed. A new trend component is finally estimated from such a series: a comparison between the old trend component and this new one can be seen in Fig. 2.

4. Discussion

As can be argued from Fig. 2, the new estimated trend appears smoother than the original one: immediate fluctuation effects due to the biological rhythm of the Po river have been removed successfully. The new trend, therefore, is able to catch clearly the underlying groundwater level, from which the health of its state can be evaluated more accurately. This model estimation is the starting point in order to provide reliable now-casting predictions. According to the geological expertise of officers involved, alarm intervals for such predictions will be discussed, aimed at building semi-automatic procedures for controlling the health of the groundwater station, in a way similar to “on line” process control in an industrial quality control scenario.

In this study, the effects of rain onto the groundwater levels have not been considered. Some preliminary investigations suggest that this predictor may not be significant for the monitored stations, because the river effect seems to be predominant. In addition, the piezometers considered refer to the particularly large hydro-graphic basins. Therefore, the effect of a single rain station, even if located next to the groundwater station, is negligible. For the analysis to become sensible, cumulative rain measurements over the whole basin should be taken into account.

References

Battaglia F. (2007) *Metodi di previsione statistica*, Springer Verlag Italia.